Proof of Theorem 226

The theorem to be proved is

$x \leq 2 \cdot \text{Half}\, x + 1$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, (x) \leq ((2 \cdot (\text{Half} x)) + 1)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, x \leq (2 \cdot (\text{Half} x)) + 1$ from H:x

1: $\neg\, \text{Parity} x = 0 \quad \vee \quad 2 \cdot (\text{Half} x) = x$ from 224;x

2: $\neg\, \text{Parity} x = 1 \quad \vee \quad (2 \cdot (\text{Half} x)) + 1 = x$ from 225;x

3: $2 \cdot (\text{Half} x) \leq (2 \cdot (\text{Half} x)) + 1$ from 71;$2 \cdot (\text{Half} x)$;1

4: $\text{Parity} x = 0 \quad \vee \quad \text{Parity} x = 1$ from 209;x

5: $x \leq x$ from 60;x

Equality substitutions:

6: $\neg\, 2 \cdot (\text{Half} x) = x \quad \vee \quad x \leq (2 \cdot (\text{Half} x)) + 1 \quad \vee \quad \neg\, x \leq (x) + 1$

7: $\neg\, 2 \cdot (\text{Half} x) = x \quad \vee \quad \neg\, 2 \cdot (\text{Half} x) \leq (2 \cdot (\text{Half} x)) + 1 \quad \vee \quad x \leq (x) + 1$

8: $\neg\, (2 \cdot (\text{Half} x)) + 1 = x \quad \vee \quad x \leq (2 \cdot (\text{Half} x)) + 1 \quad \vee \quad \neg\, x \leq x$

Inferences:

9: $\neg\, 2 \cdot (\text{Half} x) = x \quad \vee \quad \neg\, x \leq x + 1$ by
0: $\neg\, x \leq (2 \cdot (\text{Half} x)) + 1$
6: $\neg\, 2 \cdot (\text{Half} x) = x \quad \vee \quad x \leq (2 \cdot (\text{Half} x)) + 1 \quad \vee \quad \neg\, x \leq x + 1$

10: $\neg\, (2 \cdot (\text{Half} x)) + 1 = x \quad \vee \quad \neg\, x \leq x$ by
0: $\neg\, x \leq (2 \cdot (\text{Half} x)) + 1$
8: $\neg\, (2 \cdot (\text{Half} x)) + 1 = x \quad \vee \quad x \leq (2 \cdot (\text{Half} x)) + 1 \quad \vee \quad \neg\, x \leq x$

11: $\neg\, 2 \cdot (\text{Half} x) = x \quad \vee \quad x \leq x + 1$ by
3: $2 \cdot (\text{Half} x) \leq (2 \cdot (\text{Half} x)) + 1$
7: $\neg\, 2 \cdot (\text{Half} x) = x \quad \vee \quad \neg\, 2 \cdot (\text{Half} x) \leq (2 \cdot (\text{Half} x)) + 1 \quad \vee \quad x \leq x + 1$

12: $\neg\ (2 \cdot (\text{Half}x)) + 1 = x$ by

5: $x \leq x$

10: $\neg\ (2 \cdot (\text{Half}x)) + 1 = x \quad \vee \quad \neg\ x \leq x$

13: $\neg\ \text{Parity}x = 1$ by

12: $\neg\ (2 \cdot (\text{Half}x)) + 1 = x$

2: $\neg\ \text{Parity}x = 1 \quad \vee \quad (2 \cdot (\text{Half}x)) + 1 = x$

14: $\text{Parity}x = 0$ by

13: $\neg\ \text{Parity}x = 1$

4: $\text{Parity}x = 0 \quad \vee \quad \text{Parity}x = 1$

15: $2 \cdot (\text{Half}x) = x$ by

14: $\text{Parity}x = 0$

1: $\neg\ \text{Parity}x = 0 \quad \vee \quad 2 \cdot (\text{Half}x) = x$

16: $\neg\ x \leq x + 1$ by

15: $2 \cdot (\text{Half}x) = x$

9: $\neg\ 2 \cdot (\text{Half}x) = x \quad \vee \quad \neg\ x \leq x + 1$

17: $x \leq x + 1$ by

15: $2 \cdot (\text{Half}x) = x$

11: $\neg\ 2 \cdot (\text{Half}x) = x \quad \vee \quad x \leq x + 1$

18: QEA by

16: $\neg\ x \leq x + 1$

17: $x \leq x + 1$