

## Proof of Theorem 225

The theorem to be proved is

$$\text{Parity } x = 1 \rightarrow x = 2 \cdot \text{Half } x + 1$$

Suppose the theorem does not hold. Then, with the variables held fixed,

$$(H) \quad [(\text{Parity } x) = (1)] \quad \& \quad [\neg (x) = ((2 \cdot (\text{Half } x)) + 1)]$$

### Special cases of the hypothesis and previous results:

$$0: \text{Parity } x = 1 \quad \text{from } H:x$$

$$1: \neg (2 \cdot (\text{Half } x)) + 1 = x \quad \text{from } H:x$$

$$2: p_{222}(x) \quad \text{from } \underline{223};x$$

$$3: \neg p_{222}(x) \quad \vee \quad \neg \text{Parity } x = 1 \quad \vee \quad (2 \cdot (\text{Half } x)) + 1 = x \quad \text{from } \underline{222}^{\rightarrow};x$$

### Inferences:

$$4: \neg p_{222}(x) \quad \vee \quad (2 \cdot (\text{Half } x)) + 1 = x \quad \text{by}$$

$$0: \text{Parity } x = 1$$

$$3: \neg p_{222}(x) \quad \vee \quad \neg \text{Parity } x = 1 \quad \vee \quad (2 \cdot (\text{Half } x)) + 1 = x$$

$$5: \neg p_{222}(x) \quad \text{by}$$

$$1: \neg (2 \cdot (\text{Half } x)) + 1 = x$$

$$4: \neg p_{222}(x) \quad \vee \quad (2 \cdot (\text{Half } x)) + 1 = x$$

$$6: QEA \quad \text{by}$$

$$2: p_{222}(x)$$

$$5: \neg p_{222}(x)$$