

Proof of Theorem 224

The theorem to be proved is

$$\text{Parity } x = 0 \rightarrow x = 2 \cdot \text{Half } x$$

Suppose the theorem does not hold. Then, with the variables held fixed,

$$(H) \quad [(\text{Parity } x) = (0)] \quad \& \quad [\neg (x) = (2 \cdot (\text{Half } x))]$$

Special cases of the hypothesis and previous results:

- 0: $\text{Parity } x = 0$ from $H:x$
- 1: $\neg 2 \cdot (\text{Half } x) = x$ from $H:x$
- 2: $p_{222}(x)$ from [223](#); x
- 3: $\neg p_{222}(x) \vee \neg \text{Parity } x = 0 \vee 2 \cdot (\text{Half } x) = x$ from [222](#) \rightarrow ; x

Inferences:

- 4: $\neg p_{222}(x) \vee 2 \cdot (\text{Half } x) = x$ by
 - 0: $\text{Parity } x = 0$
 - 3: $\neg p_{222}(x) \vee \neg \text{Parity } x = 0 \vee 2 \cdot (\text{Half } x) = x$
- 5: $\neg p_{222}(x)$ by
 - 1: $\neg 2 \cdot (\text{Half } x) = x$
 - 4: $\neg p_{222}(x) \vee 2 \cdot (\text{Half } x) = x$
- 6: QEA by
 - 2: $p_{222}(x)$
 - 5: $\neg p_{222}(x)$