Proof of Theorem 223b

The theorem to be proved is

$p_{222}(0)$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, p_{222}(0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, p_{222}(0)$ from H

1: $S0 = 1$ from 115

2: $p_{222}(0) \;\vee\; \neg\, 2 \cdot (\text{Half}0) = 0 \;\vee\; \text{Parity}0 = 1$ from $222^{\leftarrow}$;0

3: $\text{Parity}0 = 0$ from 205

4: $\text{Half}0 = 0$ from 221

5: $2 \cdot 0 = 0$ from 99;2

6: $\neg\, S0 = 0$ from 3;0

Equality substitutions:

7: $\neg\, S0 = 1 \;\vee\; S0 = 0 \;\vee\; \neg\, 1 = 0$

8: $\neg\, \text{Parity}0 = 0 \;\vee\; \neg\, \text{Parity}0 = 1 \;\vee\; 0 = 1$

9: $\neg\, \text{Half}0 = 0 \;\vee\; 2 \cdot (\text{Half}0) = 0 \;\vee\; \neg\, 2 \cdot (0) = 0$

Inferences:

10: $\neg\, 2 \cdot (\text{Half}0) = 0 \;\vee\; \text{Parity}0 = 1$ by
0: $\neg\, p_{222}(0)$
2: $p_{222}(0) \;\vee\; \neg\, 2 \cdot (\text{Half}0) = 0 \;\vee\; \text{Parity}0 = 1$

11: $S0 = 0 \;\vee\; \neg\, 1 = 0$ by
1: $S0 = 1$
7: $\neg\, S0 = 1 \;\vee\; S0 = 0 \;\vee\; \neg\, 1 = 0$

12: $\neg\, \text{Parity}0 = 1 \;\vee\; 1 = 0$ by
3: $\text{Parity}0 = 0$
8: $\neg\, \text{Parity}0 = 0 \;\vee\; \neg\, \text{Parity}0 = 1 \;\vee\; 1 = 0$

13: $2 \cdot (\mathrm{Half}0) = 0 \quad \vee \quad \neg\, 2 \cdot 0 = 0$ by

4: $\mathrm{Half}0 = 0$

9: $\neg\, \mathrm{Half}0 = 0 \quad \vee \quad 2 \cdot (\mathrm{Half}0) = 0 \quad \vee \quad \neg\, 2 \cdot 0 = 0$

14: $2 \cdot (\mathrm{Half}0) = 0$ by

5: $2 \cdot 0 = 0$

13: $2 \cdot (\mathrm{Half}0) = 0 \quad \vee \quad \neg\, 2 \cdot 0 = 0$

15: $\neg\, 1 = 0$ by

6: $\neg\, \mathrm{S}0 = 0$

11: $\mathrm{S}0 = 0 \quad \vee \quad \neg\, 1 = 0$

16: $\mathrm{Parity}0 = 1$ by

14: $2 \cdot (\mathrm{Half}0) = 0$

10: $\neg\, 2 \cdot (\mathrm{Half}0) = 0 \quad \vee \quad \mathrm{Parity}0 = 1$

17: $\neg\, \mathrm{Parity}0 = 1$ by

15: $\neg\, 1 = 0$

12: $\neg\, \mathrm{Parity}0 = 1 \quad \vee \quad 1 = 0$

18: QEA by

16: $\mathrm{Parity}0 = 1$

17: $\neg\, \mathrm{Parity}0 = 1$