Proof of Theorem 221

The theorem to be proved is

$\text{Half } 0 = 0 \quad \& \quad \text{Half } 1 = 0 \quad \& \quad \text{Half } 2 = 1 \quad \& \quad \text{Half S2} = 1$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(\text{Half0}) = (0) \quad \vee \quad \neg\,(\text{Half1}) = (0) \quad \vee \quad \neg\,(\text{Half2}) = (1) \quad \vee \quad \neg\,(\text{Half(S2)}) = (1)]]$

Special cases of the hypothesis and previous results:

0: $\neg\,\text{Half0} = 0 \quad \vee \quad \neg\,\text{Half1} = 0 \quad \vee \quad \neg\,\text{Half2} = 1 \quad \vee \quad \neg\,\text{Half(S2)} = 1$ from H

1: $\text{S(S0)} = 2$ from 116

2: $\text{S0} = 1$ from 115

3: $\text{Half0} = 0$ from 218;0

4: $\text{C((Parity0, Half0, S(Half0)))} = \text{Half(S0)}$ from 218;0

5: $\text{C((Parity1, Half1, S(Half1)))} = \text{Half(S1)}$ from 218;1

6: $\text{C((Parity2, Half2, S(Half2)))} = \text{Half(S2)}$ from 218;2

7: $\text{Parity0} = 0$ from 208

8: $\text{Parity1} = 1$ from 208

9: $\text{Parity2} = 0$ from 208

10: $\text{C}((0, 0, 1)) = 0$ from 33;0;1;0

11: $\text{C}((\text{S0}, 0, 1)) = 1$ from 33;0;1;0

12: $\text{C}((0, 1, 2)) = 1$ from 33;1;2;0

Equality substitutions:

13: $\neg\,\text{Half0} = 0 \quad \vee \quad \neg\,\text{C((Parity0, Half0, S(Half0)))} = \text{Half(S0)} \quad \vee \quad \text{C((Parity0, 0, S(0)))} = \text{Half(S0)}$

14: $\neg\,\text{Half1} = 0 \quad \vee \quad \neg\,\text{C((Parity1, Half1, S(Half1)))} = \text{Half2} \quad \vee \quad \text{C((Parity1, 0, S(0)))} = \text{Half2}$

15: $\neg\,\text{Half2} = 1 \quad \vee \quad \neg\,\text{C((Parity2, Half2, S(Half2)))} = \text{Half(S2)} \quad \vee \quad \text{C((Parity2, 1, S(1)))} = \text{Half(S2)}$

16: $\neg\,\text{S0} = 1 \quad \vee \quad \neg\,\text{S(S0)} = 2 \quad \vee \quad \text{S}(1) = 2$

17: $\neg\ \mathrm{S0} = 1 \quad \vee \quad \neg\ \mathrm{C}((\mathrm{S0}, 0, 1)) = 1 \quad \vee \quad \mathrm{C}((1, 0, 1)) = 1$

18: $\neg\ \mathrm{S0} = 1 \quad \vee \quad \neg\ \mathrm{C}((\mathrm{Parity0}, 0, \mathrm{S0})) = \mathrm{Half}(\mathrm{S0}) \quad \vee \quad \mathrm{C}((\mathrm{Parity0}, 0, 1)) = \mathrm{Half}(1)$

19: $\neg\ \mathrm{S0} = 1 \quad \vee \quad \mathrm{C}((1, 0, \mathrm{S0})) = 1 \quad \vee \quad \neg\ \mathrm{C}((1, 0, 1)) = 1$

20: $\neg\ \mathrm{Parity0} = 0 \quad \vee \quad \mathrm{C}((\mathrm{Parity0}, 0, 1)) = 0 \quad \vee \quad \neg\ \mathrm{C}((0, 0, 1)) = 0$

21: $\neg\ \mathrm{Parity1} = 1 \quad \vee \quad \mathrm{C}((\mathrm{Parity1}, 0, \mathrm{S0})) = 1 \quad \vee \quad \neg\ \mathrm{C}((1, 0, \mathrm{S0})) = 1$

22: $\neg\ \mathrm{Parity2} = 0 \quad \vee \quad \mathrm{C}((\mathrm{Parity2}, 1, 2)) = 1 \quad \vee \quad \neg\ \mathrm{C}((0, 1, 2)) = 1$

23: $\neg\ \mathrm{C}((\mathrm{Parity2}, 1, \mathrm{S1})) = \mathrm{Half}(\mathrm{S2}) \quad \vee \quad \neg\ \mathrm{C}((\mathrm{Parity2}, 1, \mathrm{S1})) = 1 \quad \vee \quad \mathrm{Half}(\mathrm{S2}) = 1$

24: $\neg\ \mathrm{S1} = 2 \quad \vee \quad \neg\ \mathrm{C}((\mathrm{Parity1}, \mathrm{Half1}, \mathrm{S}(\mathrm{Half1}))) = \mathrm{Half}(\mathrm{S1}) \quad \vee \quad \mathrm{C}((\mathrm{Parity1}, \mathrm{Half1}, \mathrm{S}(\mathrm{Half1}))) = \mathrm{Half}(2)$

25: $\neg\ \mathrm{S1} = 2 \quad \vee \quad \mathrm{C}((\mathrm{Parity2}, 1, \mathrm{S1})) = 1 \quad \vee \quad \neg\ \mathrm{C}((\mathrm{Parity2}, 1, 2)) = 1$

26: $\neg\ \mathrm{C}((\mathrm{Parity0}, 0, 1)) = \mathrm{Half1} \quad \vee \quad \neg\ \mathrm{C}((\mathrm{Parity0}, 0, 1)) = 0 \quad \vee \quad \mathrm{Half1} = 0$

27: $\neg\ \mathrm{C}((\mathrm{Parity1}, 0, \mathrm{S0})) = \mathrm{Half2} \quad \vee \quad \neg\ \mathrm{C}((\mathrm{Parity1}, 0, \mathrm{S0})) = 1 \quad \vee \quad \mathrm{Half2} = 1$

Inferences:

28: $\neg\ \mathrm{S0} = 1 \quad \vee \quad \mathrm{S1} = 2$ by
- 1: $\mathrm{S}(\mathrm{S0}) = 2$
- 16: $\neg\ \mathrm{S0} = 1 \quad \vee \quad \neg\ \mathrm{S}(\mathrm{S0}) = 2 \quad \vee \quad \mathrm{S1} = 2$

29: $\neg\ \mathrm{C}((\mathrm{S0}, 0, 1)) = 1 \quad \vee \quad \mathrm{C}((1, 0, 1)) = 1$ by
- 2: $\mathrm{S0} = 1$
- 17: $\neg\ \mathrm{S0} = 1 \quad \vee \quad \neg\ \mathrm{C}((\mathrm{S0}, 0, 1)) = 1 \quad \vee \quad \mathrm{C}((1, 0, 1)) = 1$

30: $\neg\ \mathrm{C}((\mathrm{Parity0}, 0, \mathrm{S0})) = \mathrm{Half}(\mathrm{S0}) \quad \vee \quad \mathrm{C}((\mathrm{Parity0}, 0, 1)) = \mathrm{Half1}$ by
- 2: $\mathrm{S0} = 1$
- 18: $\neg\ \mathrm{S0} = 1 \quad \vee \quad \neg\ \mathrm{C}((\mathrm{Parity0}, 0, \mathrm{S0})) = \mathrm{Half}(\mathrm{S0}) \quad \vee \quad \mathrm{C}((\mathrm{Parity0}, 0, 1)) = \mathrm{Half1}$

31: $\mathrm{C}((1, 0, \mathrm{S0})) = 1 \quad \vee \quad \neg\ \mathrm{C}((1, 0, 1)) = 1$ by
- 2: $\mathrm{S0} = 1$
- 19: $\neg\ \mathrm{S0} = 1 \quad \vee \quad \mathrm{C}((1, 0, \mathrm{S0})) = 1 \quad \vee \quad \neg\ \mathrm{C}((1, 0, 1)) = 1$

32: $\mathrm{S1} = 2$ by
- 2: $\mathrm{S0} = 1$
- 28: $\neg\ \mathrm{S0} = 1 \quad \vee \quad \mathrm{S1} = 2$

33: $\neg\ \mathrm{Half1} = 0 \quad \vee \quad \neg\ \mathrm{Half2} = 1 \quad \vee \quad \neg\ \mathrm{Half(S2)} = 1$ by

3: $\mathrm{Half0} = 0$

0: $\neg\ \mathrm{Half0} = 0 \quad \vee \quad \neg\ \mathrm{Half1} = 0 \quad \vee \quad \neg\ \mathrm{Half2} = 1 \quad \vee \quad \neg\ \mathrm{Half(S2)} = 1$

34: $\neg\ \mathrm{C}((\mathrm{Parity0}, \mathrm{Half0}, \mathrm{S(Half0)})) = \mathrm{Half(S0)} \quad \vee \quad \mathrm{C}((\mathrm{Parity0}, 0, \mathrm{S0})) = \mathrm{Half(S0)}$ by

3: $\mathrm{Half0} = 0$

13: $\neg\ \mathrm{Half0} = 0 \quad \vee \quad \neg\ \mathrm{C}((\mathrm{Parity0}, \mathrm{Half0}, \mathrm{S(Half0)})) = \mathrm{Half(S0)} \quad \vee \quad \mathrm{C}((\mathrm{Parity0}, 0, \mathrm{S0})) = \mathrm{Half(S0)}$

35: $\mathrm{C}((\mathrm{Parity0}, 0, \mathrm{S0})) = \mathrm{Half(S0)}$ by

4: $\mathrm{C}((\mathrm{Parity0}, \mathrm{Half0}, \mathrm{S(Half0)})) = \mathrm{Half(S0)}$

34: $\neg\ \mathrm{C}((\mathrm{Parity0}, \mathrm{Half0}, \mathrm{S(Half0)})) = \mathrm{Half(S0)} \quad \vee \quad \mathrm{C}((\mathrm{Parity0}, 0, \mathrm{S0})) = \mathrm{Half(S0)}$

36: $\neg\ \mathrm{S1} = 2 \quad \vee \quad \mathrm{C}((\mathrm{Parity1}, \mathrm{Half1}, \mathrm{S(Half1)})) = \mathrm{Half2}$ by

5: $\mathrm{C}((\mathrm{Parity1}, \mathrm{Half1}, \mathrm{S(Half1)})) = \mathrm{Half(S1)}$

24: $\neg\ \mathrm{S1} = 2 \quad \vee \quad \neg\ \mathrm{C}((\mathrm{Parity1}, \mathrm{Half1}, \mathrm{S(Half1)})) = \mathrm{Half(S1)} \quad \vee \quad \mathrm{C}((\mathrm{Parity1}, \mathrm{Half1}, \mathrm{S(Half1)})) = \mathrm{Half2}$

37: $\neg\ \mathrm{Half2} = 1 \quad \vee \quad \mathrm{C}((\mathrm{Parity2}, 1, \mathrm{S1})) = \mathrm{Half(S2)}$ by

6: $\mathrm{C}((\mathrm{Parity2}, \mathrm{Half2}, \mathrm{S(Half2)})) = \mathrm{Half(S2)}$

15: $\neg\ \mathrm{Half2} = 1 \quad \vee \quad \neg\ \mathrm{C}((\mathrm{Parity2}, \mathrm{Half2}, \mathrm{S(Half2)})) = \mathrm{Half(S2)} \quad \vee \quad \mathrm{C}((\mathrm{Parity2}, 1, \mathrm{S1})) = \mathrm{Half(S2)}$

38: $\mathrm{C}((\mathrm{Parity0}, 0, 1)) = 0 \quad \vee \quad \neg\ \mathrm{C}((0, 0, 1)) = 0$ by

7: $\mathrm{Parity0} = 0$

20: $\neg\ \mathrm{Parity0} = 0 \quad \vee \quad \mathrm{C}((\mathrm{Parity0}, 0, 1)) = 0 \quad \vee \quad \neg\ \mathrm{C}((0, 0, 1)) = 0$

39: $\mathrm{C}((\mathrm{Parity1}, 0, \mathrm{S0})) = 1 \quad \vee \quad \neg\ \mathrm{C}((1, 0, \mathrm{S0})) = 1$ by

8: $\mathrm{Parity1} = 1$

21: $\neg\ \mathrm{Parity1} = 1 \quad \vee \quad \mathrm{C}((\mathrm{Parity1}, 0, \mathrm{S0})) = 1 \quad \vee \quad \neg\ \mathrm{C}((1, 0, \mathrm{S0})) = 1$

40: $\mathrm{C}((\mathrm{Parity2}, 1, 2)) = 1 \quad \vee \quad \neg\ \mathrm{C}((0, 1, 2)) = 1$ by

9: $\mathrm{Parity2} = 0$

22: $\neg\ \mathrm{Parity2} = 0 \quad \vee \quad \mathrm{C}((\mathrm{Parity2}, 1, 2)) = 1 \quad \vee \quad \neg\ \mathrm{C}((0, 1, 2)) = 1$

41: $\mathrm{C}((\mathrm{Parity0}, 0, 1)) = 0$ by

10: $\mathrm{C}((0, 0, 1)) = 0$

38: $\mathrm{C}((\mathrm{Parity0}, 0, 1)) = 0 \quad \vee \quad \neg\ \mathrm{C}((0, 0, 1)) = 0$

42: $\mathrm{C}((1, 0, 1)) = 1$ by

11: $\mathrm{C}((\mathrm{S0}, 0, 1)) = 1$

29: $\neg\ \mathrm{C}((\mathrm{S0}, 0, 1)) = 1 \quad \vee \quad \mathrm{C}((1, 0, 1)) = 1$

43: $\mathrm{C}((\mathrm{Parity2}, 1, 2)) = 1$ by

12: $\mathrm{C}((0, 1, 2)) = 1$

40: $\mathrm{C}((\mathrm{Parity2}, 1, 2)) = 1 \lor \neg\, \mathrm{C}((0, 1, 2)) = 1$

44: $\mathrm{C}((\mathrm{Parity2}, 1, \mathrm{S1})) = 1 \lor \neg\, \mathrm{C}((\mathrm{Parity2}, 1, 2)) = 1$ by

32: $\mathrm{S1} = 2$

25: $\neg\, \mathrm{S1} = 2 \lor \mathrm{C}((\mathrm{Parity2}, 1, \mathrm{S1})) = 1 \lor \neg\, \mathrm{C}((\mathrm{Parity2}, 1, 2)) = 1$

45: $\mathrm{C}((\mathrm{Parity1}, \mathrm{Half1}, \mathrm{S}(\mathrm{Half1}))) = \mathrm{Half2}$ by

32: $\mathrm{S1} = 2$

36: $\neg\, \mathrm{S1} = 2 \lor \mathrm{C}((\mathrm{Parity1}, \mathrm{Half1}, \mathrm{S}(\mathrm{Half1}))) = \mathrm{Half2}$

46: $\mathrm{C}((\mathrm{Parity0}, 0, 1)) = \mathrm{Half1}$ by

35: $\mathrm{C}((\mathrm{Parity0}, 0, \mathrm{S0})) = \mathrm{Half}(\mathrm{S0})$

30: $\neg\, \mathrm{C}((\mathrm{Parity0}, 0, \mathrm{S0})) = \mathrm{Half}(\mathrm{S0}) \lor \mathrm{C}((\mathrm{Parity0}, 0, 1)) = \mathrm{Half1}$

47: $\neg\, \mathrm{C}((\mathrm{Parity0}, 0, 1)) = \mathrm{Half1} \lor \mathrm{Half1} = 0$ by

41: $\mathrm{C}((\mathrm{Parity0}, 0, 1)) = 0$

26: $\neg\, \mathrm{C}((\mathrm{Parity0}, 0, 1)) = \mathrm{Half1} \lor \neg\, \mathrm{C}((\mathrm{Parity0}, 0, 1)) = 0 \lor \mathrm{Half1} = 0$

48: $\mathrm{C}((1, 0, \mathrm{S0})) = 1$ by

42: $\mathrm{C}((1, 0, 1)) = 1$

31: $\mathrm{C}((1, 0, \mathrm{S0})) = 1 \lor \neg\, \mathrm{C}((1, 0, 1)) = 1$

49: $\mathrm{C}((\mathrm{Parity2}, 1, \mathrm{S1})) = 1$ by

43: $\mathrm{C}((\mathrm{Parity2}, 1, 2)) = 1$

44: $\mathrm{C}((\mathrm{Parity2}, 1, \mathrm{S1})) = 1 \lor \neg\, \mathrm{C}((\mathrm{Parity2}, 1, 2)) = 1$

50: $\neg\, \mathrm{Half1} = 0 \lor \mathrm{C}((\mathrm{Parity1}, 0, \mathrm{S0})) = \mathrm{Half2}$ by

45: $\mathrm{C}((\mathrm{Parity1}, \mathrm{Half1}, \mathrm{S}(\mathrm{Half1}))) = \mathrm{Half2}$

14: $\neg\, \mathrm{Half1} = 0 \lor \neg\, \mathrm{C}((\mathrm{Parity1}, \mathrm{Half1}, \mathrm{S}(\mathrm{Half1}))) = \mathrm{Half2} \lor \mathrm{C}((\mathrm{Parity1}, 0, \mathrm{S0})) = \mathrm{Half2}$

51: $\mathrm{Half1} = 0$ by

46: $\mathrm{C}((\mathrm{Parity0}, 0, 1)) = \mathrm{Half1}$

47: $\neg\, \mathrm{C}((\mathrm{Parity0}, 0, 1)) = \mathrm{Half1} \lor \mathrm{Half1} = 0$

52: $\mathrm{C}((\mathrm{Parity1}, 0, \mathrm{S0})) = 1$ by

48: $\mathrm{C}((1, 0, \mathrm{S0})) = 1$

39: $\mathrm{C}((\mathrm{Parity1}, 0, \mathrm{S0})) = 1 \lor \neg\, \mathrm{C}((1, 0, \mathrm{S0})) = 1$

53: $\neg\, \mathrm{C}((\mathrm{Parity2}, 1, \mathrm{S1})) = \mathrm{Half}(\mathrm{S2}) \lor \mathrm{Half}(\mathrm{S2}) = 1$ by

49: $\mathrm{C}((\mathrm{Parity2}, 1, \mathrm{S1})) = 1$

23: $\neg\ \mathrm{C}((\mathrm{Parity2}, 1, \mathrm{S1})) = \mathrm{Half(S2)} \quad \vee \quad \neg\ \mathrm{C}((\mathrm{Parity2}, 1, \mathrm{S1})) = 1 \quad \vee \quad \mathrm{Half(S2)} = 1$

54: $\neg\ \mathrm{Half2} = 1 \quad \vee \quad \neg\ \mathrm{Half(S2)} = 1$ by

51: $\mathrm{Half1} = 0$

33: $\neg\ \mathrm{Half1} = 0 \quad \vee \quad \neg\ \mathrm{Half2} = 1 \quad \vee \quad \neg\ \mathrm{Half(S2)} = 1$

55: $\mathrm{C}((\mathrm{Parity1}, 0, \mathrm{S0})) = \mathrm{Half2}$ by

51: $\mathrm{Half1} = 0$

50: $\neg\ \mathrm{Half1} = 0 \quad \vee \quad \mathrm{C}((\mathrm{Parity1}, 0, \mathrm{S0})) = \mathrm{Half2}$

56: $\neg\ \mathrm{C}((\mathrm{Parity1}, 0, \mathrm{S0})) = \mathrm{Half2} \quad \vee \quad \mathrm{Half2} = 1$ by

52: $\mathrm{C}((\mathrm{Parity1}, 0, \mathrm{S0})) = 1$

27: $\neg\ \mathrm{C}((\mathrm{Parity1}, 0, \mathrm{S0})) = \mathrm{Half2} \quad \vee \quad \neg\ \mathrm{C}((\mathrm{Parity1}, 0, \mathrm{S0})) = 1 \quad \vee \quad \mathrm{Half2} = 1$

57: $\mathrm{Half2} = 1$ by

55: $\mathrm{C}((\mathrm{Parity1}, 0, \mathrm{S0})) = \mathrm{Half2}$

56: $\neg\ \mathrm{C}((\mathrm{Parity1}, 0, \mathrm{S0})) = \mathrm{Half2} \quad \vee \quad \mathrm{Half2} = 1$

58: $\mathrm{C}((\mathrm{Parity2}, 1, \mathrm{S1})) = \mathrm{Half(S2)}$ by

57: $\mathrm{Half2} = 1$

37: $\neg\ \mathrm{Half2} = 1 \quad \vee \quad \mathrm{C}((\mathrm{Parity2}, 1, \mathrm{S1})) = \mathrm{Half(S2)}$

59: $\neg\ \mathrm{Half(S2)} = 1$ by

57: $\mathrm{Half2} = 1$

54: $\neg\ \mathrm{Half2} = 1 \quad \vee \quad \neg\ \mathrm{Half(S2)} = 1$

60: $\mathrm{Half(S2)} = 1$ by

58: $\mathrm{C}((\mathrm{Parity2}, 1, \mathrm{S1})) = \mathrm{Half(S2)}$

53: $\neg\ \mathrm{C}((\mathrm{Parity2}, 1, \mathrm{S1})) = \mathrm{Half(S2)} \quad \vee \quad \mathrm{Half(S2)} = 1$

61: QEA by

59: $\neg\ \mathrm{Half(S2)} = 1$

60: $\mathrm{Half(S2)} = 1$