Proof of Theorem 219

The theorem to be proved is

Parity $x = 0 \quad \rightarrow \quad$ Half Sx = Half x

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $\quad$ [[(Parity$x$) = (0)] $\quad$ & $\quad$ [¬ (Half(Sx)) = (Half$x$)]]

Special cases of the hypothesis and previous results:

0: Parity$x$ = 0 $\quad$ from $\quad$ H:x

1: ¬ Half(Sx) = Half$x$ $\quad$ from $\quad$ H:x

2: C((Parity$x$, Half$x$, S(Half$x$))) = Half(Sx) $\quad$ from $\quad$ 218;x

3: C((0, Half$x$, S(Half$x$))) = Half$x$ $\quad$ from $\quad$ 33;Half$x$;S(Half$x$)

Equality substitutions:

4: ¬ Parity$x$ = 0 ∨ ¬ C((Parity$x$, Half$x$, S(Half$x$))) = Half(Sx) ∨ C((0, Half$x$, S(Half$x$))) = Half(Sx)

5: ¬ C((0, Half$x$, S(Half$x$))) = Half$x$ ∨ ¬ C((0, Half$x$, S(Half$x$))) = Half(Sx) ∨ Half$x$ = Half(Sx)

Inferences:

6: ¬ C((Parity$x$, Half$x$, S(Half$x$))) = Half(Sx) ∨ C((0, Half$x$, S(Half$x$))) = Half(Sx) by

0: Parity$x$ = 0

4: ¬ Parity$x$ = 0 ∨ ¬ C((Parity$x$, Half$x$, S(Half$x$))) = Half(Sx) ∨ C((0, Half$x$, S(Half$x$))) = Half(Sx)

7: ¬ C((0, Half$x$, S(Half$x$))) = Half$x$ ∨ ¬ C((0, Half$x$, S(Half$x$))) = Half(Sx) $\quad$ by

1: ¬ Half(Sx) = Half$x$

5: ¬ C((0, Half$x$, S(Half$x$))) = Half$x$ ∨ ¬ C((0, Half$x$, S(Half$x$))) = Half(Sx) ∨ Half(Sx) = Half$x$

8: C((0, Half$x$, S(Half$x$))) = Half(Sx) $\quad$ by

2: C((Parity$x$, Half$x$, S(Half$x$))) = Half(Sx)

6: ¬ C((Parity$x$, Half$x$, S(Half$x$))) = Half(Sx) ∨ C((0, Half$x$, S(Half$x$))) = Half(Sx)

9: $\neg\, \mathrm{C}((0, \mathrm{Half}x, \mathrm{S}(\mathrm{Half}x))) = \mathrm{Half}(\mathrm{S}x)$ by

3: $\mathrm{C}((0, \mathrm{Half}x, \mathrm{S}(\mathrm{Half}x))) = \mathrm{Half}x$

7: $\neg\, \mathrm{C}((0, \mathrm{Half}x, \mathrm{S}(\mathrm{Half}x))) = \mathrm{Half}x \quad \vee \quad \neg\, \mathrm{C}((0, \mathrm{Half}x, \mathrm{S}(\mathrm{Half}x))) = \mathrm{Half}(\mathrm{S}x)$

10: QEA by

8: $\mathrm{C}((0, \mathrm{Half}x, \mathrm{S}(\mathrm{Half}x))) = \mathrm{Half}(\mathrm{S}x)$

9: $\neg\, \mathrm{C}((0, \mathrm{Half}x, \mathrm{S}(\mathrm{Half}x))) = \mathrm{Half}(\mathrm{S}x)$