Proof of Theorem 217

The theorem to be proved is

$x \oplus \underline{0} \neq y \oplus \underline{1}$ ⋆

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x \oplus \underline{0}) = (y \oplus \underline{1})]]$

Special cases of the hypothesis and previous results:

0: $y \oplus \underline{1} = x \oplus \underline{0}$ from H:x:y

1: $\mathrm{S}0 = 1$ from 115

2: $\mathrm{Parity}(\mathrm{R}(x \oplus \underline{0})) = 0$ from 215;x

3: $\mathrm{Parity}(\mathrm{R}(y \oplus \underline{1})) = 1$ from 216;y

4: $\neg\ \mathrm{S}0 = 0$ from 3;0

Equality substitutions:

5: $\neg\ y \oplus \underline{1} = x \oplus \underline{0} \quad \vee \quad \mathrm{Parity}(\mathrm{R}(y \oplus \underline{1})) = 0 \quad \vee \quad \neg\ \mathrm{Parity}(\mathrm{R}(x \oplus \underline{0})) = 0$

6: $\neg\ \mathrm{S}0 = 1 \quad \vee \quad \mathrm{S}0 = 0 \quad \vee \quad \neg\ 1 = 0$

7: $\neg\ \mathrm{Parity}(\mathrm{R}(y \oplus \underline{1})) = 1 \quad \vee \quad \neg\ \mathrm{Parity}(\mathrm{R}(y \oplus \underline{1})) = 0 \quad \vee \quad 1 = 0$

Inferences:

8: $\mathrm{Parity}(\mathrm{R}(y \oplus \underline{1})) = 0 \quad \vee \quad \neg\ \mathrm{Parity}(\mathrm{R}(x \oplus \underline{0})) = 0$ by
0: $y \oplus \underline{1} = x \oplus \underline{0}$
5: $\neg\ y \oplus \underline{1} = x \oplus \underline{0} \quad \vee \quad \mathrm{Parity}(\mathrm{R}(y \oplus \underline{1})) = 0 \quad \vee \quad \neg\ \mathrm{Parity}(\mathrm{R}(x \oplus \underline{0})) = 0$

9: $\mathrm{S}0 = 0 \quad \vee \quad \neg\ 1 = 0$ by
1: $\mathrm{S}0 = 1$
6: $\neg\ \mathrm{S}0 = 1 \quad \vee \quad \mathrm{S}0 = 0 \quad \vee \quad \neg\ 1 = 0$

10: $\mathrm{Parity}(\mathrm{R}(y \oplus \underline{1})) = 0$ by
2: $\mathrm{Parity}(\mathrm{R}(x \oplus \underline{0})) = 0$
8: $\mathrm{Parity}(\mathrm{R}(y \oplus \underline{1})) = 0 \quad \vee \quad \neg\ \mathrm{Parity}(\mathrm{R}(x \oplus \underline{0})) = 0$

11: $\neg\, \mathrm{Parity}(\mathrm{R}(y \oplus \underline{1})) = 0 \quad \vee \quad 1 = 0$ by

3: $\mathrm{Parity}(\mathrm{R}(y \oplus \underline{1})) = 1$

7: $\neg\, \mathrm{Parity}(\mathrm{R}(y \oplus \underline{1})) = 1 \quad \vee \quad \neg\, \mathrm{Parity}(\mathrm{R}(y \oplus \underline{1})) = 0 \quad \vee \quad 1 = 0$

12: $\neg\, 1 = 0$ by

4: $\neg\, \mathrm{S}0 = 0$

9: $\mathrm{S}0 = 0 \quad \vee \quad \neg\, 1 = 0$

13: $1 = 0$ by

10: $\mathrm{Parity}(\mathrm{R}(y \oplus \underline{1})) = 0$

11: $\neg\, \mathrm{Parity}(\mathrm{R}(y \oplus \underline{1})) = 0 \quad \vee \quad 1 = 0$

14: QEA by

12: $\neg\, 1 = 0$

13: $1 = 0$