Proof of Theorem 216

The theorem to be proved is

Parity $\mathrm{R}(x \oplus \underline{1}) = 1$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, (\mathrm{Parity}(\mathrm{R}(x \oplus \underline{1}))) = (1)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, \mathrm{Parity}(\mathrm{R}(x \oplus \underline{1})) = 1$ from H:x

1: $((\mathrm{R}x) \cdot (\mathrm{Q}\underline{1})) + (\mathrm{R}\underline{1}) = \mathrm{R}(x \oplus \underline{1})$ from 180;x;$\underline{1}$

2: $\mathrm{Q}\underline{1} = 2$ from 192

3: $\mathrm{R}\underline{1} = 1$ from 192

4: $\mathrm{Parity}(((\mathrm{R}x) \cdot 2) + 1) = \mathrm{Parity}1$ from 212;Rx;1

5: $\mathrm{Parity}1 = 1$ from 208

Equality substitutions:

6: $\neg\, \mathrm{Q}\underline{1} = 2 \;\vee\; \neg\, ((\mathrm{R}x) \cdot (\mathrm{Q}\underline{1})) + (\mathrm{R}\underline{1}) = \mathrm{R}(x \oplus \underline{1}) \;\vee\; ((\mathrm{R}x) \cdot (2)) + (\mathrm{R}\underline{1}) = \mathrm{R}(x \oplus \underline{1})$

7: $\neg\, \mathrm{R}\underline{1} = 1 \;\vee\; \neg\, ((\mathrm{R}x) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}(x \oplus \underline{1}) \;\vee\; ((\mathrm{R}x) \cdot 2) + (1) = \mathrm{R}(x \oplus \underline{1})$

8: $\neg\, \mathrm{Parity}1 = 1 \;\vee\; \neg\, \mathrm{Parity}(\mathrm{R}(x \oplus \underline{1})) = \mathrm{Parity}1 \;\vee\; \mathrm{Parity}(\mathrm{R}(x \oplus \underline{1})) = 1$

9: $\neg\, ((\mathrm{R}x) \cdot 2) + 1 = \mathrm{R}(x \oplus \underline{1}) \;\vee\; \neg\, \mathrm{Parity}(((\mathrm{R}x) \cdot 2) + 1) = \mathrm{Parity}1 \;\vee\; \mathrm{Parity}(\mathrm{R}(x \oplus \underline{1})) = \mathrm{Parity}1$

Inferences:

10: $\neg\, \mathrm{Parity}1 = 1 \;\vee\; \neg\, \mathrm{Parity}(\mathrm{R}(x \oplus \underline{1})) = \mathrm{Parity}1$ by

0: $\neg\, \mathrm{Parity}(\mathrm{R}(x \oplus \underline{1})) = 1$

8: $\neg\, \mathrm{Parity}1 = 1 \;\vee\; \neg\, \mathrm{Parity}(\mathrm{R}(x \oplus \underline{1})) = \mathrm{Parity}1 \;\vee\; \mathrm{Parity}(\mathrm{R}(x \oplus \underline{1})) = 1$

11: $\neg\, \mathrm{Q}\underline{1} = 2 \;\vee\; ((\mathrm{R}x) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}(x \oplus \underline{1})$ by

1: $((\mathrm{R}x) \cdot (\mathrm{Q}\underline{1})) + (\mathrm{R}\underline{1}) = \mathrm{R}(x \oplus \underline{1})$

6: $\neg\, \mathrm{Q}\underline{1} = 2 \;\vee\; \neg\, ((\mathrm{R}x) \cdot (\mathrm{Q}\underline{1})) + (\mathrm{R}\underline{1}) = \mathrm{R}(x \oplus \underline{1}) \;\vee\; ((\mathrm{R}x) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}(x \oplus \underline{1})$

12: $((\mathrm{R}x) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}(x \oplus \underline{1})$ by

2: $\mathrm{Q}\underline{1} = 2$

11: $\neg\ \mathrm{Q}\underline{1} = 2 \quad \vee \quad ((\mathrm{R}x) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}(x \oplus \underline{1})$

13: $\neg\ ((\mathrm{R}x) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}(x \oplus \underline{1}) \quad \vee \quad ((\mathrm{R}x) \cdot 2) + 1 = \mathrm{R}(x \oplus \underline{1})$ by

3: $\mathrm{R}\underline{1} = 1$

7: $\neg\ \mathrm{R}\underline{1} = 1 \quad \vee \quad \neg\ ((\mathrm{R}x) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}(x \oplus \underline{1}) \quad \vee \quad ((\mathrm{R}x) \cdot 2) + 1 = \mathrm{R}(x \oplus \underline{1})$

14: $\neg\ ((\mathrm{R}x) \cdot 2) + 1 = \mathrm{R}(x \oplus \underline{1}) \quad \vee \quad \mathrm{Parity}(\mathrm{R}(x \oplus \underline{1})) = \mathrm{Parity}1$ by

4: $\mathrm{Parity}(((\mathrm{R}x) \cdot 2) + 1) = \mathrm{Parity}1$

9: $\neg\ ((\mathrm{R}x) \cdot 2) + 1 = \mathrm{R}(x \oplus \underline{1}) \quad \vee \quad \neg\ \mathrm{Parity}(((\mathrm{R}x) \cdot 2) + 1) = \mathrm{Parity}1 \quad \vee$ $\mathrm{Parity}(\mathrm{R}(x \oplus \underline{1})) = \mathrm{Parity}1$

15: $\neg\ \mathrm{Parity}(\mathrm{R}(x \oplus \underline{1})) = \mathrm{Parity}1$ by

5: $\mathrm{Parity}1 = 1$

10: $\neg\ \mathrm{Parity}1 = 1 \quad \vee \quad \neg\ \mathrm{Parity}(\mathrm{R}(x \oplus \underline{1})) = \mathrm{Parity}1$

16: $((\mathrm{R}x) \cdot 2) + 1 = \mathrm{R}(x \oplus \underline{1})$ by

12: $((\mathrm{R}x) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}(x \oplus \underline{1})$

13: $\neg\ ((\mathrm{R}x) \cdot 2) + (\mathrm{R}\underline{1}) = \mathrm{R}(x \oplus \underline{1}) \quad \vee \quad ((\mathrm{R}x) \cdot 2) + 1 = \mathrm{R}(x \oplus \underline{1})$

17: $\neg\ ((\mathrm{R}x) \cdot 2) + 1 = \mathrm{R}(x \oplus \underline{1})$ by

15: $\neg\ \mathrm{Parity}(\mathrm{R}(x \oplus \underline{1})) = \mathrm{Parity}1$

14: $\neg\ ((\mathrm{R}x) \cdot 2) + 1 = \mathrm{R}(x \oplus \underline{1}) \quad \vee \quad \mathrm{Parity}(\mathrm{R}(x \oplus \underline{1})) = \mathrm{Parity}1$

18: QEA by

16: $((\mathrm{R}x) \cdot 2) + 1 = \mathrm{R}(x \oplus \underline{1})$

17: $\neg\ ((\mathrm{R}x) \cdot 2) + 1 = \mathrm{R}(x \oplus \underline{1})$