Proof of Theorem 215

The theorem to be proved is

Parity $\mathrm{R}(x \oplus \underline{0}) = 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, (\mathrm{Parity}(\mathrm{R}(x \oplus \underline{0}))) = (0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, \mathrm{Parity}(\mathrm{R}(x \oplus \underline{0})) = 0$ from H:x

1: $((\mathrm{R}x) \cdot (\mathrm{Q}\underline{0})) + (\mathrm{R}\underline{0}) = \mathrm{R}(x \oplus \underline{0})$ from 180;x;$\underline{0}$

2: $\mathrm{Q}\underline{0} = 2$ from 191

3: $\mathrm{R}\underline{0} = 0$ from 191

4: $\mathrm{Parity}(((\mathrm{R}x) \cdot 2) + 0) = \mathrm{Parity}0$ from 212;Rx;0

5: $\mathrm{Parity}0 = 0$ from 205

Equality substitutions:

6: $\neg\, \mathrm{Q}\underline{0} = 2 \;\vee\; \neg\, ((\mathrm{R}x) \cdot (\mathrm{Q}\underline{0})) + (\mathrm{R}\underline{0}) = \mathrm{R}(x \oplus \underline{0}) \;\vee\; ((\mathrm{R}x) \cdot (2)) + (\mathrm{R}\underline{0}) = \mathrm{R}(x \oplus \underline{0})$

7: $\neg\, \mathrm{R}\underline{0} = 0 \;\vee\; \neg\, ((\mathrm{R}x) \cdot 2) + (\mathrm{R}\underline{0}) = \mathrm{R}(x \oplus \underline{0}) \;\vee\; ((\mathrm{R}x) \cdot 2) + (0) = \mathrm{R}(x \oplus \underline{0})$

8: $\neg\, \mathrm{Parity}0 = 0 \;\vee\; \neg\, \mathrm{Parity}(\mathrm{R}(x \oplus \underline{0})) = \mathrm{Parity}0 \;\vee\; \mathrm{Parity}(\mathrm{R}(x \oplus \underline{0})) = 0$

9: $\neg\, ((\mathrm{R}x) \cdot 2) + 0 = \mathrm{R}(x \oplus \underline{0}) \;\vee\; \neg\, \mathrm{Parity}(((\mathrm{R}x) \cdot 2) + 0) = \mathrm{Parity}0 \;\vee\; \mathrm{Parity}(\mathrm{R}(x \oplus \underline{0})) = \mathrm{Parity}0$

Inferences:

10: $\neg\, \mathrm{Parity}0 = 0 \;\vee\; \neg\, \mathrm{Parity}(\mathrm{R}(x \oplus \underline{0})) = \mathrm{Parity}0$ by

0: $\neg\, \mathrm{Parity}(\mathrm{R}(x \oplus \underline{0})) = 0$

8: $\neg\, \mathrm{Parity}0 = 0 \;\vee\; \neg\, \mathrm{Parity}(\mathrm{R}(x \oplus \underline{0})) = \mathrm{Parity}0 \;\vee\; \mathrm{Parity}(\mathrm{R}(x \oplus \underline{0})) = 0$

11: $\neg\, \mathrm{Q}\underline{0} = 2 \;\vee\; ((\mathrm{R}x) \cdot 2) + (\mathrm{R}\underline{0}) = \mathrm{R}(x \oplus \underline{0})$ by

1: $((\mathrm{R}x) \cdot (\mathrm{Q}\underline{0})) + (\mathrm{R}\underline{0}) = \mathrm{R}(x \oplus \underline{0})$

6: $\neg\, \mathrm{Q}\underline{0} = 2 \;\vee\; \neg\, ((\mathrm{R}x) \cdot (\mathrm{Q}\underline{0})) + (\mathrm{R}\underline{0}) = \mathrm{R}(x \oplus \underline{0}) \;\vee\; ((\mathrm{R}x) \cdot 2) + (\mathrm{R}\underline{0}) = \mathrm{R}(x \oplus \underline{0})$

12: $((\mathrm{R}x)\cdot 2) + (\mathrm{R}\underline{0}) = \mathrm{R}(x \oplus \underline{0})$ by

2: $\mathrm{Q}\underline{0} = 2$

11: $\neg\ \mathrm{Q}\underline{0} = 2 \quad \vee \quad ((\mathrm{R}x)\cdot 2) + (\mathrm{R}\underline{0}) = \mathrm{R}(x \oplus \underline{0})$

13: $\neg\ ((\mathrm{R}x)\cdot 2) + (\mathrm{R}\underline{0}) = \mathrm{R}(x \oplus \underline{0}) \quad \vee \quad ((\mathrm{R}x)\cdot 2) + 0 = \mathrm{R}(x \oplus \underline{0})$ by

3: $\mathrm{R}\underline{0} = 0$

7: $\neg\ \mathrm{R}\underline{0} = 0 \quad \vee \quad \neg\ ((\mathrm{R}x)\cdot 2) + (\mathrm{R}\underline{0}) = \mathrm{R}(x \oplus \underline{0}) \quad \vee \quad ((\mathrm{R}x)\cdot 2) + 0 = \mathrm{R}(x \oplus \underline{0})$

14: $\neg\ ((\mathrm{R}x)\cdot 2) + 0 = \mathrm{R}(x \oplus \underline{0}) \quad \vee \quad \mathrm{Parity}(\mathrm{R}(x \oplus \underline{0})) = \mathrm{Parity}0$ by

4: $\mathrm{Parity}(((\mathrm{R}x)\cdot 2) + 0) = \mathrm{Parity}0$

9: $\neg\ ((\mathrm{R}x)\cdot 2) + 0 = \mathrm{R}(x \oplus \underline{0}) \quad \vee \quad \neg\ \mathrm{Parity}(((\mathrm{R}x)\cdot 2) + 0) = \mathrm{Parity}0 \quad \vee$ $\mathrm{Parity}(\mathrm{R}(x \oplus \underline{0})) = \mathrm{Parity}0$

15: $\neg\ \mathrm{Parity}(\mathrm{R}(x \oplus \underline{0})) = \mathrm{Parity}0$ by

5: $\mathrm{Parity}0 = 0$

10: $\neg\ \mathrm{Parity}0 = 0 \quad \vee \quad \neg\ \mathrm{Parity}(\mathrm{R}(x \oplus \underline{0})) = \mathrm{Parity}0$

16: $((\mathrm{R}x)\cdot 2) + 0 = \mathrm{R}(x \oplus \underline{0})$ by

12: $((\mathrm{R}x)\cdot 2) + (\mathrm{R}\underline{0}) = \mathrm{R}(x \oplus \underline{0})$

13: $\neg\ ((\mathrm{R}x)\cdot 2) + (\mathrm{R}\underline{0}) = \mathrm{R}(x \oplus \underline{0}) \quad \vee \quad ((\mathrm{R}x)\cdot 2) + 0 = \mathrm{R}(x \oplus \underline{0})$

17: $\neg\ ((\mathrm{R}x)\cdot 2) + 0 = \mathrm{R}(x \oplus \underline{0})$ by

15: $\neg\ \mathrm{Parity}(\mathrm{R}(x \oplus \underline{0})) = \mathrm{Parity}0$

14: $\neg\ ((\mathrm{R}x)\cdot 2) + 0 = \mathrm{R}(x \oplus \underline{0}) \quad \vee \quad \mathrm{Parity}(\mathrm{R}(x \oplus \underline{0})) = \mathrm{Parity}0$

18: QEA by

16: $((\mathrm{R}x)\cdot 2) + 0 = \mathrm{R}(x \oplus \underline{0})$

17: $\neg\ ((\mathrm{R}x)\cdot 2) + 0 = \mathrm{R}(x \oplus \underline{0})$