Proof of Theorem 214

The theorem to be proved is

Parity$(x \cdot 2 + 1) = 1$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\ (\text{Parity}((x \cdot 2) + 1)) = (1)]]$

Special cases of the hypothesis and previous results:

0: $\neg\ \text{Parity}((x \cdot 2) + 1) = 1$ from H:x

1: $\text{Parity}((x \cdot 2) + 1) = \text{Parity1}$ from 212;x;1

2: $\text{Parity1} = 1$ from 208

Equality substitutions:

3: $\neg\ \text{Parity}((x \cdot 2) + 1) = \text{Parity1} \quad \vee \quad \text{Parity}((x \cdot 2) + 1) = 1 \quad \vee \quad \neg\ \text{Parity1} = 1$

Inferences:

4: $\neg\ \text{Parity}((x \cdot 2) + 1) = \text{Parity1} \quad \vee \quad \neg\ \text{Parity1} = 1$ by
0: $\neg\ \text{Parity}((x \cdot 2) + 1) = 1$
3: $\neg\ \text{Parity}((x \cdot 2) + 1) = \text{Parity1} \quad \vee \quad \text{Parity}((x \cdot 2) + 1) = 1 \quad \vee \quad \neg\ \text{Parity1} = 1$

5: $\neg\ \text{Parity1} = 1$ by
1: $\text{Parity}((x \cdot 2) + 1) = \text{Parity1}$
4: $\neg\ \text{Parity}((x \cdot 2) + 1) = \text{Parity1} \quad \vee \quad \neg\ \text{Parity1} = 1$

6: QEA by
2: $\text{Parity1} = 1$
5: $\neg\ \text{Parity1} = 1$