Proof of Theorem 213

The theorem to be proved is

$\text{Parity}(x \cdot 2) = 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\ (\text{Parity}(x \cdot 2)) = (0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\ \text{Parity}(x \cdot 2) = 0$ from H:x

1: $\text{Parity}((x \cdot 2) + 0) = \text{Parity}0$ from 212;x;0

2: $\text{Parity}0 = 0$ from 208

3: $(x \cdot 2) + 0 = x \cdot 2$ from 12;$x \cdot 2$

Equality substitutions:

4: $\neg\ \text{Parity}0 = 0 \quad \vee \quad \neg\ \text{Parity}(x \cdot 2) = \text{Parity}0 \quad \vee \quad \text{Parity}(x \cdot 2) = 0$

5: $\neg\ (x \cdot 2) + 0 = x \cdot 2 \quad \vee \quad \neg\ \text{Parity}((x \cdot 2) + 0) = \text{Parity}0 \quad \vee \quad \text{Parity}(x \cdot 2) = \text{Parity}0$

Inferences:

6: $\neg\ \text{Parity}0 = 0 \quad \vee \quad \neg\ \text{Parity}(x \cdot 2) = \text{Parity}0$ by

0: $\neg\ \text{Parity}(x \cdot 2) = 0$

4: $\neg\ \text{Parity}0 = 0 \quad \vee \quad \neg\ \text{Parity}(x \cdot 2) = \text{Parity}0 \quad \vee \quad \text{Parity}(x \cdot 2) = 0$

7: $\neg\ (x \cdot 2) + 0 = x \cdot 2 \quad \vee \quad \text{Parity}(x \cdot 2) = \text{Parity}0$ by

1: $\text{Parity}((x \cdot 2) + 0) = \text{Parity}0$

5: $\neg\ (x \cdot 2) + 0 = x \cdot 2 \quad \vee \quad \neg\ \text{Parity}((x \cdot 2) + 0) = \text{Parity}0 \quad \vee \quad \text{Parity}(x \cdot 2) = \text{Parity}0$

8: $\neg\ \text{Parity}(x \cdot 2) = \text{Parity}0$ by

2: $\text{Parity}0 = 0$

6: $\neg\ \text{Parity}0 = 0 \quad \vee \quad \neg\ \text{Parity}(x \cdot 2) = \text{Parity}0$

9: $\text{Parity}(x \cdot 2) = \text{Parity}0$ by

3: $(x \cdot 2) + 0 = x \cdot 2$

7: $\neg\ (x \cdot 2) + 0 = x \cdot 2 \quad \vee \quad \text{Parity}(x \cdot 2) = \text{Parity}0$

10: QEA by

8: $\neg\ \text{Parity}(x \cdot 2) = \text{Parity}0$

9: $\text{Parity}(x \cdot 2) = \text{Parity}0$