Proof of Theorem 212

The theorem to be proved is

$\text{Parity}(x \cdot 2 + y) = \text{Parity}\, y$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, (\text{Parity}((x \cdot 2) + y)) = (\text{Parity}y)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, \text{Parity}((x \cdot 2) + y) = \text{Parity}y$ from H:x:y

1: $\text{Parity}2 = 0$ from 208

2: $2 \cdot x = x \cdot 2$ from 105;x;2

3: $\neg\, \text{Parity}2 = 0 \lor \text{Parity}(2 \cdot x) = 0$ from 211;2;x

4: $\neg\, \text{Parity}(2 \cdot x) = 0 \lor \text{Parity}((2 \cdot x) + y) = \text{Parity}y$ from 210;$2 \cdot x$;y

Equality substitutions:

5: $\neg\, 2 \cdot x = x \cdot 2 \lor \neg\, \text{Parity}((2 \cdot x) + y) = \text{Parity}y \lor \text{Parity}((x \cdot 2) + y) = \text{Parity}y$

Inferences:

6: $\neg\, 2 \cdot x = x \cdot 2 \lor \neg\, \text{Parity}((2 \cdot x) + y) = \text{Parity}y$ by

0: $\neg\, \text{Parity}((x \cdot 2) + y) = \text{Parity}y$

5: $\neg\, 2 \cdot x = x \cdot 2 \lor \neg\, \text{Parity}((2 \cdot x) + y) = \text{Parity}y \lor \text{Parity}((x \cdot 2) + y) = \text{Parity}y$

7: $\text{Parity}(2 \cdot x) = 0$ by

1: $\text{Parity}2 = 0$

3: $\neg\, \text{Parity}2 = 0 \lor \text{Parity}(2 \cdot x) = 0$

8: $\neg\, \text{Parity}((2 \cdot x) + y) = \text{Parity}y$ by

2: $2 \cdot x = x \cdot 2$

6: $\neg\, 2 \cdot x = x \cdot 2 \lor \neg\, \text{Parity}((2 \cdot x) + y) = \text{Parity}y$

9: $\text{Parity}((2 \cdot x) + y) = \text{Parity}y$ by

7: $\text{Parity}(2 \cdot x) = 0$

4: $\neg\, \text{Parity}(2 \cdot x) = 0 \lor \text{Parity}((2 \cdot x) + y) = \text{Parity}y$

10: QEA by

8: $\neg\, \text{Parity}((2 \cdot x) + y) = \text{Parity}y$

9: $\text{Parity}((2 \cdot x) + y) = \text{Parity}y$