Proof of Theorem 211i

The theorem to be proved is

$[\text{Parity}\, x = 0 \quad \rightarrow \quad \text{Parity}(x \cdot y) = 0] \quad \rightarrow \quad [\text{Parity}\, x = 0 \quad \rightarrow \quad \text{Parity}(x \cdot \text{S}y) = 0]$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\ (\text{Parity}x) = (0) \quad \vee \quad (\text{Parity}(x \cdot y)) = (0)] \quad \& \quad [(\text{Parity}x) = (0)] \quad \& \quad [\neg\ (\text{Parity}(x \cdot (\text{S}y))) = (0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\ \text{Parity}x = 0 \quad \vee \quad \text{Parity}(x \cdot y) = 0$ from H:x:y

1: $\text{Parity}x = 0$ from H:x:y

2: $\neg\ \text{Parity}(x \cdot (\text{S}y)) = 0$ from H:x:y

3: $x + (x \cdot y) = x \cdot (\text{S}y)$ from 99;x;y

4: $\neg\ \text{Parity}x = 0 \quad \vee \quad \text{Parity}(x + (x \cdot y)) = \text{Parity}(x \cdot y)$ from 210;x;$x \cdot y$

Equality substitutions:

5: $\neg\ \text{Parity}(x \cdot y) = 0 \quad \vee \quad \neg\ \text{Parity}(x + (x \cdot y)) = \text{Parity}(x \cdot y) \quad \vee \quad \text{Parity}(x + (x \cdot y)) = 0$

6: $\neg\ x + (x \cdot y) = x \cdot (\text{S}y) \quad \vee \quad \neg\ \text{Parity}(x + (x \cdot y)) = 0 \quad \vee \quad \text{Parity}(x \cdot (\text{S}y)) = 0$

Inferences:

7: $\text{Parity}(x \cdot y) = 0$ by

1: $\text{Parity}x = 0$

0: $\neg\ \text{Parity}x = 0 \quad \vee \quad \text{Parity}(x \cdot y) = 0$

8: $\text{Parity}(x + (x \cdot y)) = \text{Parity}(x \cdot y)$ by

1: $\text{Parity}x = 0$

4: $\neg\ \text{Parity}x = 0 \quad \vee \quad \text{Parity}(x + (x \cdot y)) = \text{Parity}(x \cdot y)$

9: $\neg\ x + (x \cdot y) = x \cdot (\text{S}y) \quad \vee \quad \neg\ \text{Parity}(x + (x \cdot y)) = 0$ by

2: $\neg\ \text{Parity}(x \cdot (\text{S}y)) = 0$

6: $\neg\ x + (x \cdot y) = x \cdot (\text{S}y) \quad \vee \quad \neg\ \text{Parity}(x + (x \cdot y)) = 0 \quad \vee \quad \text{Parity}(x \cdot (\text{S}y)) = 0$

10: $\neg\ \text{Parity}(x + (x \cdot y)) = 0$ by

3: $x + (x \cdot y) = x \cdot (\text{S}y)$

9: $\neg\ x + (x \cdot y) = x \cdot (\text{S}y) \quad \vee \quad \neg\ \text{Parity}(x + (x \cdot y)) = 0$

11: $\neg\, \text{Parity}(x + (x \cdot y)) = \text{Parity}(x \cdot y) \quad \vee \quad \text{Parity}(x + (x \cdot y)) = 0$ by

7: $\text{Parity}(x \cdot y) = 0$

5: $\neg\, \text{Parity}(x \cdot y) = 0 \quad \vee \quad \neg\, \text{Parity}(x + (x \cdot y)) = \text{Parity}(x \cdot y) \quad \vee \quad \text{Parity}(x + (x \cdot y)) = 0$

12: $\text{Parity}(x + (x \cdot y)) = 0$ by

8: $\text{Parity}(x + (x \cdot y)) = \text{Parity}(x \cdot y)$

11: $\neg\, \text{Parity}(x + (x \cdot y)) = \text{Parity}(x \cdot y) \quad \vee \quad \text{Parity}(x + (x \cdot y)) = 0$

13: QEA by

10: $\neg\, \text{Parity}(x + (x \cdot y)) = 0$

12: $\text{Parity}(x + (x \cdot y)) = 0$