Proof of Theorem 211b

The theorem to be proved is

$\text{Parity}\, x = 0 \quad \rightarrow \quad \text{Parity}(x \cdot 0) = 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(\text{Parity} x) = (0)] \quad \& \quad [\neg\, (\text{Parity}(x \cdot 0)) = (0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, \text{Parity}(x \cdot 0) = 0$ from H:x

1: $x \cdot 0 = 0$ from 100;x

2: $\text{Parity}0 = 0$ from 205

Equality substitutions:

3: $\neg\, x \cdot 0 = 0 \quad \vee \quad \text{Parity}(x \cdot 0) = 0 \quad \vee \quad \neg\, \text{Parity}(0) = 0$

Inferences:

4: $\neg\, x \cdot 0 = 0 \quad \vee \quad \neg\, \text{Parity}0 = 0$ by
0: $\neg\, \text{Parity}(x \cdot 0) = 0$
3: $\neg\, x \cdot 0 = 0 \quad \vee \quad \text{Parity}(x \cdot 0) = 0 \quad \vee \quad \neg\, \text{Parity}0 = 0$

5: $\neg\, \text{Parity}0 = 0$ by
1: $x \cdot 0 = 0$
4: $\neg\, x \cdot 0 = 0 \quad \vee \quad \neg\, \text{Parity}0 = 0$

6: QEA by
2: $\text{Parity}0 = 0$
5: $\neg\, \text{Parity}0 = 0$