Proof of Theorem 210i

The theorem to be proved is

$[\text{Parity}\, x = 0 \quad \rightarrow \quad \text{Parity}(x+y) = \text{Parity}\, y] \quad \rightarrow \quad [\text{Parity}\, x = 0 \quad \rightarrow \quad \text{Parity}(x+\text{S}y) = \text{Parity}\, \text{S}y]$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\ (\text{Parity}x) = (0) \quad \vee \quad (\text{Parity}(x+y)) = (\text{Parity}y)] \quad \& \quad [(\text{Parity}x) = (0)] \quad \& \quad [\neg\ (\text{Parity}(x+(\text{S}y))) = (\text{Parity}(\text{S}y))]]$

Special cases of the hypothesis and previous results:

0: $\neg\ \text{Parity}x = 0 \quad \vee \quad \text{Parity}(x+y) = \text{Parity}y$ from H:x:y

1: $\text{Parity}x = 0$ from H:x:y

2: $\neg\ \text{Parity}(x+(\text{S}y)) = \text{Parity}(\text{S}y)$ from H:x:y

3: $\text{S}(x+y) = x+(\text{S}y)$ from 12;x;y

4: $\text{C}((\text{Parity}(x+y), 1, 0)) = \text{Parity}(\text{S}(x+y))$ from 205;$x+y$

5: $\text{C}((\text{Parity}y, 1, 0)) = \text{Parity}(\text{S}y)$ from 205;y

Equality substitutions:

6: $\neg\ \text{Parity}(x+y) = \text{Parity}y \quad \vee \quad \neg\ \text{C}((\text{Parity}(x+y), 1, 0)) = \text{Parity}(x+(\text{S}y)) \quad \vee \quad \text{C}((\text{Parity}y, 1, 0)) = \text{Parity}(x+(\text{S}y))$

7: $\neg\ \text{S}(x+y) = x+(\text{S}y) \quad \vee \quad \neg\ \text{C}((\text{Parity}(x+y), 1, 0)) = \text{Parity}(\text{S}(x+y)) \quad \vee \quad \text{C}((\text{Parity}(x+y), 1, 0)) = \text{Parity}(x+(\text{S}y))$

8: $\neg\ \text{C}((\text{Parity}y, 1, 0)) = \text{Parity}(\text{S}y) \quad \vee \quad \neg\ \text{C}((\text{Parity}y, 1, 0)) = \text{Parity}(x+(\text{S}y)) \quad \vee \quad \text{Parity}(\text{S}y) = \text{Parity}(x+(\text{S}y))$

Inferences:

9: $\text{Parity}(x+y) = \text{Parity}y$ by
1: $\text{Parity}x = 0$
0: $\neg\ \text{Parity}x = 0 \quad \vee \quad \text{Parity}(x+y) = \text{Parity}y$

10: $\neg\ \text{C}((\text{Parity}y, 1, 0)) = \text{Parity}(\text{S}y) \quad \vee \quad \neg\ \text{C}((\text{Parity}y, 1, 0)) = \text{Parity}(x+(\text{S}y))$ by
2: $\neg\ \text{Parity}(x+(\text{S}y)) = \text{Parity}(\text{S}y)$

8: $\neg$ C((Parity$y$, 1, 0)) = Parity(Sy) $\vee$ $\neg$ C((Parity$y$, 1, 0)) = Parity(x + (Sy)) $\vee$ Parity(x + (Sy)) = Parity(Sy)

11: $\neg$ C((Parity($x + y$), 1, 0)) = Parity(S($x + y$)) $\vee$ C((Parity($x + y$), 1, 0)) = Parity(x + (Sy)) by

3: S($x + y$) = x + (Sy)

7: $\neg$ S($x + y$) = x + (Sy) $\vee$ $\neg$ C((Parity($x + y$), 1, 0)) = Parity(S($x + y$)) $\vee$ C((Parity($x + y$), 1, 0)) = Parity(x + (Sy))

12: C((Parity($x + y$), 1, 0)) = Parity(x + (Sy)) by

4: C((Parity($x + y$), 1, 0)) = Parity(S($x + y$))

11: $\neg$ C((Parity($x + y$), 1, 0)) = Parity(S($x + y$)) $\vee$ C((Parity($x + y$), 1, 0)) = Parity(x + (Sy))

13: $\neg$ C((Parity$y$, 1, 0)) = Parity(x + (Sy)) by

5: C((Parity$y$, 1, 0)) = Parity(Sy)

10: $\neg$ C((Parity$y$, 1, 0)) = Parity(Sy) $\vee$ $\neg$ C((Parity$y$, 1, 0)) = Parity(x + (Sy))

14: $\neg$ C((Parity($x + y$), 1, 0)) = Parity(x + (Sy)) $\vee$ C((Parity$y$, 1, 0)) = Parity(x + (Sy)) by

9: Parity($x + y$) = Parity$y$

6: $\neg$ Parity($x + y$) = Parity$y$ $\vee$ $\neg$ C((Parity($x + y$), 1, 0)) = Parity(x + (Sy)) $\vee$ C((Parity$y$, 1, 0)) = Parity(x + (Sy))

15: C((Parity$y$, 1, 0)) = Parity(x + (Sy)) by

12: C((Parity($x + y$), 1, 0)) = Parity(x + (Sy))

14: $\neg$ C((Parity($x + y$), 1, 0)) = Parity(x + (Sy)) $\vee$ C((Parity$y$, 1, 0)) = Parity(x + (Sy))

16: QEA by

13: $\neg$ C((Parity$y$, 1, 0)) = Parity(x + (Sy))

15: C((Parity$y$, 1, 0)) = Parity(x + (Sy))