Proof of Theorem 210b

The theorem to be proved is

$\text{Parity}\,x = 0 \quad \to \quad \text{Parity}(x+0) = \text{Parity}\,0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(\text{Parity}x) = (0)] \quad \& \quad [\neg\,(\text{Parity}(x+0)) = (\text{Parity}0)]]$

Special cases of the hypothesis and previous results:

0: $\text{Parity}x = 0$ from H:x

1: $\neg\,\text{Parity}(x+0) = \text{Parity}0$ from H:x

2: $\text{Parity}0 = 0$ from 208

3: $x + 0 = x$ from 12;x

Equality substitutions:

4: $\neg\,\text{Parity}x = 0 \quad \vee \quad \text{Parity}0 = \text{Parity}x \quad \vee \quad \neg\,\text{Parity}0 = 0$

5: $\neg\, x+0 = x \quad \vee \quad \text{Parity}(x+0) = \text{Parity}0 \quad \vee \quad \neg\,\text{Parity}(x) = \text{Parity}0$

Inferences:

6: $\text{Parity}0 = \text{Parity}x \quad \vee \quad \neg\,\text{Parity}0 = 0$ by

0: $\text{Parity}x = 0$

4: $\neg\,\text{Parity}x = 0 \quad \vee \quad \text{Parity}0 = \text{Parity}x \quad \vee \quad \neg\,\text{Parity}0 = 0$

7: $\neg\, x+0 = x \quad \vee \quad \neg\,\text{Parity}0 = \text{Parity}x$ by

1: $\neg\,\text{Parity}(x+0) = \text{Parity}0$

5: $\neg\, x+0 = x \quad \vee \quad \text{Parity}(x+0) = \text{Parity}0 \quad \vee \quad \neg\,\text{Parity}0 = \text{Parity}x$

8: $\text{Parity}0 = \text{Parity}x$ by

2: $\text{Parity}0 = 0$

6: $\text{Parity}0 = \text{Parity}x \quad \vee \quad \neg\,\text{Parity}0 = 0$

9: $\neg\,\text{Parity}0 = \text{Parity}x$ by

3: $x + 0 = x$

7: $\neg\, x+0 = x \quad \vee \quad \neg\,\text{Parity}0 = \text{Parity}x$

10: QEA by

8: $\text{Parity}0 = \text{Parity}x$

9: $\neg\,\text{Parity}0 = \text{Parity}x$