Proof of Theorem 21

The theorem to be proved is

$\mathrm{S}x - x \neq 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[((\mathrm{S}x) - x) = (0)]]$

Special cases of the hypothesis and previous results:

0: $(\mathrm{S}x) - x = 0$ from H:x

1: $(\mathrm{S}x) - x = \mathrm{S}0$ from 20;x

2: $\neg\ \mathrm{S}0 = 0$ from 3;0

Equality substitutions:

3: $\neg\ (\mathrm{S}x) - x = 0 \quad \vee \quad \neg\ (\mathrm{S}x) - x = \mathrm{S}0 \quad \vee \quad 0 = \mathrm{S}0$

Inferences:

4: $\neg\ (\mathrm{S}x) - x = \mathrm{S}0 \quad \vee \quad \mathrm{S}0 = 0$ by
- 0: $(\mathrm{S}x) - x = 0$
- 3: $\neg\ (\mathrm{S}x) - x = 0 \quad \vee \quad \neg\ (\mathrm{S}x) - x = \mathrm{S}0 \quad \vee \quad \mathrm{S}0 = 0$

5: $\mathrm{S}0 = 0$ by
- 1: $(\mathrm{S}x) - x = \mathrm{S}0$
- 4: $\neg\ (\mathrm{S}x) - x = \mathrm{S}0 \quad \vee \quad \mathrm{S}0 = 0$

6: QEA by
- 2: $\neg\ \mathrm{S}0 = 0$
- 5: $\mathrm{S}0 = 0$