Proof of Theorem 20i

The theorem to be proved is

$Sx - x = S0 \quad \rightarrow \quad SSx - Sx = S0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[((Sx) - x) = (S0)] \quad \& \quad [\neg\, ((S(Sx)) - (Sx)) = (S0)]]$

Special cases of the hypothesis and previous results:

0: $(Sx) - x = S0$ from H:x

1: $\neg\, (S(Sx)) - (Sx) = S0$ from H:x

2: $(S(Sx)) - (Sx) = (Sx) - x$ from 18;Sx;x

Equality substitutions:

3: $\neg\, (Sx) - x = S0 \quad \vee \quad \neg\, (S(Sx)) - (Sx) = (Sx) - x \quad \vee \quad (S(Sx)) - (Sx) = S0$

Inferences:

4: $\neg\, (S(Sx)) - (Sx) = (Sx) - x \quad \vee \quad (S(Sx)) - (Sx) = S0$ by
  0: $(Sx) - x = S0$
  3: $\neg\, (Sx) - x = S0 \quad \vee \quad \neg\, (S(Sx)) - (Sx) = (Sx) - x \quad \vee \quad (S(Sx)) - (Sx) = S0$

5: $\neg\, (S(Sx)) - (Sx) = (Sx) - x$ by
  1: $\neg\, (S(Sx)) - (Sx) = S0$
  4: $\neg\, (S(Sx)) - (Sx) = (Sx) - x \quad \vee \quad (S(Sx)) - (Sx) = S0$

6: QEA by
  2: $(S(Sx)) - (Sx) = (Sx) - x$
  5: $\neg\, (S(Sx)) - (Sx) = (Sx) - x$