Proof of Theorem 209

The theorem to be proved is

$\text{Parity}\, x = 0 \quad \vee \quad \text{Parity}\, x = 1$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\ (\text{Parity}x) = (0)] \quad \& \quad [\neg\ (\text{Parity}x) = (1)]]$

Special cases of the hypothesis and previous results:

0: $\neg\ \text{Parity}x = 0$ from H:x

1: $\neg\ \text{Parity}x = 1$ from H:x

2: $\text{S}0 = 1$ from 115

3: $\text{Parity}0 = 0$ from 205;x

4: $\text{C}((\text{Parity}(\text{P}x), 1, 0)) = \text{Parity}(\text{S}(\text{P}x))$ from 205;Px

5: $0 = x \quad \vee \quad \text{S}(\text{P}x) = x$ from 22;x

6: $\text{C}((\text{Parity}(\text{P}x), \text{S}0, 0)) = 0 \quad \vee \quad \text{C}((\text{Parity}(\text{P}x), \text{S}0, 0)) = \text{S}0$ from 39;Parity(Px)

Equality substitutions:

7: $\neg\ \text{S}0 = 1 \quad \vee \quad \neg\ \text{C}((\text{Parity}(\text{P}x), \text{S}0, 0)) = 0 \quad \vee \quad \text{C}((\text{Parity}(\text{P}x), 1, 0)) = 0$

8: $\neg\ \text{S}0 = 1 \quad \vee \quad \neg\ \text{C}((\text{Parity}(\text{P}x), \text{S}0, 0)) = \text{S}0 \quad \vee \quad \text{C}((\text{Parity}(\text{P}x), 1, 0)) = 1$

9: $\neg\ \text{C}((\text{Parity}(\text{P}x), 1, 0)) = \text{Parity}(\text{S}(\text{P}x)) \quad \vee \quad \neg\ \text{C}((\text{Parity}(\text{P}x), 1, 0)) = 0 \quad \vee \quad \text{Parity}(\text{S}(\text{P}x)) = 0$

10: $\neg\ \text{C}((\text{Parity}(\text{P}x), 1, 0)) = \text{Parity}(\text{S}(\text{P}x)) \quad \vee \quad \neg\ \text{C}((\text{Parity}(\text{P}x), 1, 0)) = 1 \quad \vee \quad \text{Parity}(\text{S}(\text{P}x)) = 1$

11: $\neg\ \text{S}(\text{P}x) = x \quad \vee \quad \neg\ \text{Parity}(\text{S}(\text{P}x)) = 0 \quad \vee \quad \text{Parity}(x) = 0$

12: $\neg\ \text{S}(\text{P}x) = x \quad \vee \quad \neg\ \text{Parity}(\text{S}(\text{P}x)) = 1 \quad \vee \quad \text{Parity}(x) = 1$

13: $\neg\ x = 0 \quad \vee \quad \text{Parity}(x) = 0 \quad \vee \quad \neg\ \text{Parity}(0) = 0$

Inferences:

14: $\neg\, \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \neg\, \mathrm{Parity}(\mathrm{S}(\mathrm{P}x)) = 0$ by

0: $\neg\, \mathrm{Parity}x = 0$

11: $\neg\, \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \neg\, \mathrm{Parity}(\mathrm{S}(\mathrm{P}x)) = 0 \quad \vee \quad \mathrm{Parity}x = 0$

15: $\neg\, 0 = x \quad \vee \quad \neg\, \mathrm{Parity}0 = 0$ by

0: $\neg\, \mathrm{Parity}x = 0$

13: $\neg\, 0 = x \quad \vee \quad \mathrm{Parity}x = 0 \quad \vee \quad \neg\, \mathrm{Parity}0 = 0$

16: $\neg\, \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \neg\, \mathrm{Parity}(\mathrm{S}(\mathrm{P}x)) = 1$ by

1: $\neg\, \mathrm{Parity}x = 1$

12: $\neg\, \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \neg\, \mathrm{Parity}(\mathrm{S}(\mathrm{P}x)) = 1 \quad \vee \quad \mathrm{Parity}x = 1$

17: $\neg\, \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), \mathrm{S}0, 0)) = 0 \quad \vee \quad \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), 1, 0)) = 0$ by

2: $\mathrm{S}0 = 1$

7: $\neg\, \mathrm{S}0 = 1 \quad \vee \quad \neg\, \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), \mathrm{S}0, 0)) = 0 \quad \vee \quad \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), 1, 0)) = 0$

18: $\neg\, \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), \mathrm{S}0, 0)) = \mathrm{S}0 \quad \vee \quad \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), 1, 0)) = 1$ by

2: $\mathrm{S}0 = 1$

8: $\neg\, \mathrm{S}0 = 1 \quad \vee \quad \neg\, \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), \mathrm{S}0, 0)) = \mathrm{S}0 \quad \vee \quad \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), 1, 0)) = 1$

19: $\neg\, 0 = x$ by

3: $\mathrm{Parity}0 = 0$

15: $\neg\, 0 = x \quad \vee \quad \neg\, \mathrm{Parity}0 = 0$

20: $\neg\, \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), 1, 0)) = 0 \quad \vee \quad \mathrm{Parity}(\mathrm{S}(\mathrm{P}x)) = 0$ by

4: $\mathrm{C}((\mathrm{Parity}(\mathrm{P}x), 1, 0)) = \mathrm{Parity}(\mathrm{S}(\mathrm{P}x))$

9: $\neg\, \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), 1, 0)) = \mathrm{Parity}(\mathrm{S}(\mathrm{P}x)) \quad \vee \quad \neg\, \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), 1, 0)) = 0 \quad \vee \quad \mathrm{Parity}(\mathrm{S}(\mathrm{P}x)) = 0$

21: $\neg\, \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), 1, 0)) = 1 \quad \vee \quad \mathrm{Parity}(\mathrm{S}(\mathrm{P}x)) = 1$ by

4: $\mathrm{C}((\mathrm{Parity}(\mathrm{P}x), 1, 0)) = \mathrm{Parity}(\mathrm{S}(\mathrm{P}x))$

10: $\neg\, \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), 1, 0)) = \mathrm{Parity}(\mathrm{S}(\mathrm{P}x)) \quad \vee \quad \neg\, \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), 1, 0)) = 1 \quad \vee \quad \mathrm{Parity}(\mathrm{S}(\mathrm{P}x)) = 1$

22: $\mathrm{S}(\mathrm{P}x) = x$ by

19: $\neg\, 0 = x$

5: $0 = x \quad \vee \quad \mathrm{S}(\mathrm{P}x) = x$

23: $\neg\, \mathrm{Parity}(\mathrm{S}(\mathrm{P}x)) = 0$ by

22: $\mathrm{S}(\mathrm{P}x) = x$

14: $\neg\, \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \neg\, \mathrm{Parity}(\mathrm{S}(\mathrm{P}x)) = 0$

24: $\neg\ \mathrm{Parity}(\mathrm{S}(\mathrm{P}x)) = 1$ by

22: $\mathrm{S}(\mathrm{P}x) = x$

16: $\neg\ \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \neg\ \mathrm{Parity}(\mathrm{S}(\mathrm{P}x)) = 1$

25: $\neg\ \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), 1, 0)) = 0$ by

23: $\neg\ \mathrm{Parity}(\mathrm{S}(\mathrm{P}x)) = 0$

20: $\neg\ \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), 1, 0)) = 0 \quad \vee \quad \mathrm{Parity}(\mathrm{S}(\mathrm{P}x)) = 0$

26: $\neg\ \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), 1, 0)) = 1$ by

24: $\neg\ \mathrm{Parity}(\mathrm{S}(\mathrm{P}x)) = 1$

21: $\neg\ \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), 1, 0)) = 1 \quad \vee \quad \mathrm{Parity}(\mathrm{S}(\mathrm{P}x)) = 1$

27: $\neg\ \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), \mathrm{S}0, 0)) = 0$ by

25: $\neg\ \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), 1, 0)) = 0$

17: $\neg\ \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), \mathrm{S}0, 0)) = 0 \quad \vee \quad \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), 1, 0)) = 0$

28: $\neg\ \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), \mathrm{S}0, 0)) = \mathrm{S}0$ by

26: $\neg\ \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), 1, 0)) = 1$

18: $\neg\ \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), \mathrm{S}0, 0)) = \mathrm{S}0 \quad \vee \quad \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), 1, 0)) = 1$

29: $\mathrm{C}((\mathrm{Parity}(\mathrm{P}x), \mathrm{S}0, 0)) = \mathrm{S}0$ by

27: $\neg\ \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), \mathrm{S}0, 0)) = 0$

6: $\mathrm{C}((\mathrm{Parity}(\mathrm{P}x), \mathrm{S}0, 0)) = 0 \quad \vee \quad \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), \mathrm{S}0, 0)) = \mathrm{S}0$

30: QEA by

28: $\neg\ \mathrm{C}((\mathrm{Parity}(\mathrm{P}x), \mathrm{S}0, 0)) = \mathrm{S}0$

29: $\mathrm{C}((\mathrm{Parity}(\mathrm{P}x), \mathrm{S}0, 0)) = \mathrm{S}0$