Proof of Theorem 208

The theorem to be proved is

$\text{Parity}\,0 = 0 \quad \& \quad \text{Parity}\,1 = 1 \quad \& \quad \text{Parity}\,2 = 0 \quad \& \quad \text{Parity}\,\epsilon = 0 \quad \& \quad \text{Parity}\,\underline{0} = 1 \quad \& \quad \text{Parity}\,\underline{1} = 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(\text{Parity}0) = (0) \quad \vee \quad \neg\,(\text{Parity}1) = (1) \quad \vee \quad \neg\,(\text{Parity}2) = (0) \quad \vee \quad \neg\,(\text{Parity}\epsilon) = (0) \quad \vee \quad \neg\,(\text{Parity}\underline{0}) = (1) \quad \vee \quad \neg\,(\text{Parity}\underline{1}) = (0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\ \text{Parity}0 = 0 \quad \vee \quad \neg\ \text{Parity}1 = 1 \quad \vee \quad \neg\ \text{Parity}2 = 0 \quad \vee \quad \neg\ \text{Parity}\epsilon = 0 \quad \vee \quad \neg\ \text{Parity}\underline{0} = 1 \quad \vee \quad \neg\ \text{Parity}\underline{1} = 0$ from H

1: $\text{S}(\text{S}0) = 2$ from 116

2: $\text{S}0 = 1$ from 115

3: $\text{Parity}0 = 0$ from 205;0

4: $\text{C}((\text{Parity}0, 1, 0)) = \text{Parity}(\text{S}0)$ from 205;0

5: $\text{C}((\text{Parity}1, 1, 0)) = \text{Parity}(\text{S}1)$ from 205;1

6: $\epsilon = 0$ from 185

7: $\underline{0} = 1$ from 186

8: $\underline{1} = 2$ from 187

9: $\text{C}((0, 1, 0)) = 1$ from 33;1;0;0

10: $\text{C}((\text{S}0, 1, 0)) = 0$ from 33;1;0;0

Equality substitutions:

11: $\neg\ \text{Parity}0 = 0 \quad \vee \quad \text{C}((\text{Parity}0, 1, 0)) = 1 \quad \vee \quad \neg\ \text{C}((0, 1, 0)) = 1$

12: $\neg\ \text{Parity}1 = 1 \quad \vee \quad \text{C}((\text{Parity}1, 1, 0)) = 0 \quad \vee \quad \neg\ \text{C}((1, 1, 0)) = 0$

13: $\neg\ \text{S}0 = 1 \quad \vee \quad \neg\ \text{S}(\text{S}0) = 2 \quad \vee \quad \text{S}(1) = 2$

14: $\neg\ \text{S}0 = 1 \quad \vee \quad \neg\ \text{C}((\text{Parity}0, 1, 0)) = \text{Parity}(\text{S}0) \quad \vee \quad \text{C}((\text{Parity}0, 1, 0)) = \text{Parity}(1)$

15: $\neg\ \text{S}0 = 1 \quad \vee \quad \neg\ \text{C}((\text{S}0, 1, 0)) = 0 \quad \vee \quad \text{C}((1, 1, 0)) = 0$

16: $\neg\ \epsilon = 0 \quad \vee \quad \text{Parity}\epsilon = 0 \quad \vee \quad \neg\ \text{Parity}0 = 0$

17: $\neg\ \underline{0} = 1 \quad \vee \quad \text{Parity}\underline{0} = 1 \quad \vee \quad \neg\ \text{Parity}1 = 1$

18: $\neg\ \underline{1} = 2 \quad \vee \quad \text{Parity}\underline{1} = 0 \quad \vee \quad \neg\ \text{Parity}2 = 0$

19: $\neg\ \text{S}1 = 2 \quad \vee \quad \neg\ \text{C}((\text{Parity}1, 1, 0)) = \text{Parity}(\text{S}1) \quad \vee \quad \text{C}((\text{Parity}1, 1, 0)) = \text{Parity}(2)$

20: $\neg\ \text{C}((\text{Parity}0, 1, 0)) = \text{Parity}1 \quad \vee \quad \neg\ \text{C}((\text{Parity}0, 1, 0)) = 1 \quad \vee \quad \text{Parity}1 = 1$

21: $\neg\ \text{C}((\text{Parity}1, 1, 0)) = \text{Parity}2 \quad \vee \quad \neg\ \text{C}((\text{Parity}1, 1, 0)) = 0 \quad \vee \quad \text{Parity}2 = 0$

Inferences:

22: $\neg\ \text{S}0 = 1 \quad \vee \quad \text{S}1 = 2$ by
1: $\text{S}(\text{S}0) = 2$
13: $\neg\ \text{S}0 = 1 \quad \vee \quad \neg\ \text{S}(\text{S}0) = 2 \quad \vee \quad \text{S}1 = 2$

23: $\neg\ \text{C}((\text{Parity}0, 1, 0)) = \text{Parity}(\text{S}0) \quad \vee \quad \text{C}((\text{Parity}0, 1, 0)) = \text{Parity}1$ by
2: $\text{S}0 = 1$
14: $\neg\ \text{S}0 = 1 \quad \vee \quad \neg\ \text{C}((\text{Parity}0, 1, 0)) = \text{Parity}(\text{S}0) \quad \vee \quad \text{C}((\text{Parity}0, 1, 0)) = \text{Parity}1$

24: $\neg\ \text{C}((\text{S}0, 1, 0)) = 0 \quad \vee \quad \text{C}((1, 1, 0)) = 0$ by
2: $\text{S}0 = 1$
15: $\neg\ \text{S}0 = 1 \quad \vee \quad \neg\ \text{C}((\text{S}0, 1, 0)) = 0 \quad \vee \quad \text{C}((1, 1, 0)) = 0$

25: $\text{S}1 = 2$ by
2: $\text{S}0 = 1$
22: $\neg\ \text{S}0 = 1 \quad \vee \quad \text{S}1 = 2$

26: $\neg\ \text{Parity}1 = 1 \quad \vee \quad \neg\ \text{Parity}2 = 0 \quad \vee \quad \neg\ \text{Parity}\epsilon = 0 \quad \vee \quad \neg\ \text{Parity}\underline{0} = 1 \quad \vee \quad \neg\ \text{Parity}\underline{1} = 0$ by
3: $\text{Parity}0 = 0$
0: $\neg\ \text{Parity}0 = 0 \quad \vee \quad \neg\ \text{Parity}1 = 1 \quad \vee \quad \neg\ \text{Parity}2 = 0 \quad \vee \quad \neg\ \text{Parity}\epsilon = 0 \quad \vee \quad \neg\ \text{Parity}\underline{0} = 1 \quad \vee \quad \neg\ \text{Parity}\underline{1} = 0$

27: $\text{C}((\text{Parity}0, 1, 0)) = 1 \quad \vee \quad \neg\ \text{C}((0, 1, 0)) = 1$ by
3: $\text{Parity}0 = 0$
11: $\neg\ \text{Parity}0 = 0 \quad \vee \quad \text{C}((\text{Parity}0, 1, 0)) = 1 \quad \vee \quad \neg\ \text{C}((0, 1, 0)) = 1$

28: $\neg\ \epsilon = 0 \quad \vee \quad \text{Parity}\epsilon = 0$ by
3: $\text{Parity}0 = 0$
16: $\neg\ \epsilon = 0 \quad \vee \quad \text{Parity}\epsilon = 0 \quad \vee \quad \neg\ \text{Parity}0 = 0$

29: $\mathrm{C}((\mathrm{Parity0}, 1, 0)) = \mathrm{Parity1}$ by

4: $\mathrm{C}((\mathrm{Parity0}, 1, 0)) = \mathrm{Parity}(\mathrm{S0})$

23: $\neg\, \mathrm{C}((\mathrm{Parity0}, 1, 0)) = \mathrm{Parity}(\mathrm{S0}) \quad \vee \quad \mathrm{C}((\mathrm{Parity0}, 1, 0)) = \mathrm{Parity1}$

30: $\neg\, \mathrm{S1} = 2 \quad \vee \quad \mathrm{C}((\mathrm{Parity1}, 1, 0)) = \mathrm{Parity2}$ by

5: $\mathrm{C}((\mathrm{Parity1}, 1, 0)) = \mathrm{Parity}(\mathrm{S1})$

19: $\neg\, \mathrm{S1} = 2 \quad \vee \quad \neg\, \mathrm{C}((\mathrm{Parity1}, 1, 0)) = \mathrm{Parity}(\mathrm{S1}) \quad \vee \quad \mathrm{C}((\mathrm{Parity1}, 1, 0)) = \mathrm{Parity2}$

31: $\mathrm{Parity}\epsilon = 0$ by

6: $\epsilon = 0$

28: $\neg\, \epsilon = 0 \quad \vee \quad \mathrm{Parity}\epsilon = 0$

32: $\mathrm{Parity}\underline{0} = 1 \quad \vee \quad \neg\, \mathrm{Parity1} = 1$ by

7: $\underline{0} = 1$

17: $\neg\, \underline{0} = 1 \quad \vee \quad \mathrm{Parity}\underline{0} = 1 \quad \vee \quad \neg\, \mathrm{Parity1} = 1$

33: $\mathrm{Parity}\underline{1} = 0 \quad \vee \quad \neg\, \mathrm{Parity2} = 0$ by

8: $\underline{1} = 2$

18: $\neg\, \underline{1} = 2 \quad \vee \quad \mathrm{Parity}\underline{1} = 0 \quad \vee \quad \neg\, \mathrm{Parity2} = 0$

34: $\mathrm{C}((\mathrm{Parity0}, 1, 0)) = 1$ by

9: $\mathrm{C}((0, 1, 0)) = 1$

27: $\mathrm{C}((\mathrm{Parity0}, 1, 0)) = 1 \quad \vee \quad \neg\, \mathrm{C}((0, 1, 0)) = 1$

35: $\mathrm{C}((1, 1, 0)) = 0$ by

10: $\mathrm{C}((\mathrm{S0}, 1, 0)) = 0$

24: $\neg\, \mathrm{C}((\mathrm{S0}, 1, 0)) = 0 \quad \vee \quad \mathrm{C}((1, 1, 0)) = 0$

36: $\mathrm{C}((\mathrm{Parity1}, 1, 0)) = \mathrm{Parity2}$ by

25: $\mathrm{S1} = 2$

30: $\neg\, \mathrm{S1} = 2 \quad \vee \quad \mathrm{C}((\mathrm{Parity1}, 1, 0)) = \mathrm{Parity2}$

37: $\neg\, \mathrm{C}((\mathrm{Parity0}, 1, 0)) = 1 \quad \vee \quad \mathrm{Parity1} = 1$ by

29: $\mathrm{C}((\mathrm{Parity0}, 1, 0)) = \mathrm{Parity1}$

20: $\neg\, \mathrm{C}((\mathrm{Parity0}, 1, 0)) = \mathrm{Parity1} \quad \vee \quad \neg\, \mathrm{C}((\mathrm{Parity0}, 1, 0)) = 1 \quad \vee \quad \mathrm{Parity1} = 1$

38: $\neg\, \mathrm{Parity1} = 1 \quad \vee \quad \neg\, \mathrm{Parity2} = 0 \quad \vee \quad \neg\, \mathrm{Parity}\underline{0} = 1 \quad \vee \quad \neg\, \mathrm{Parity}\underline{1} = 0$ by

31: $\mathrm{Parity}\epsilon = 0$

26: $\neg\, \mathrm{Parity1} = 1 \quad \vee \quad \neg\, \mathrm{Parity2} = 0 \quad \vee \quad \neg\, \mathrm{Parity}\epsilon = 0 \quad \vee \quad \neg\, \mathrm{Parity}\underline{0} = 1$
$\vee \quad \neg\, \mathrm{Parity}\underline{1} = 0$

39: $\mathrm{Parity1} = 1$ by

34: $\mathrm{C}((\mathrm{Parity0}, 1, 0)) = 1$

37: $\neg\ \mathrm{C}((\mathrm{Parity0}, 1, 0)) = 1 \quad \vee \quad \mathrm{Parity1} = 1$

40: $\neg\ \mathrm{Parity1} = 1 \quad \vee \quad \mathrm{C}((\mathrm{Parity1}, 1, 0)) = 0$ by

35: $\mathrm{C}((1, 1, 0)) = 0$

12: $\neg\ \mathrm{Parity1} = 1 \quad \vee \quad \mathrm{C}((\mathrm{Parity1}, 1, 0)) = 0 \quad \vee \quad \neg\ \mathrm{C}((1, 1, 0)) = 0$

41: $\neg\ \mathrm{C}((\mathrm{Parity1}, 1, 0)) = 0 \quad \vee \quad \mathrm{Parity2} = 0$ by

36: $\mathrm{C}((\mathrm{Parity1}, 1, 0)) = \mathrm{Parity2}$

21: $\neg\ \mathrm{C}((\mathrm{Parity1}, 1, 0)) = \mathrm{Parity2} \quad \vee \quad \neg\ \mathrm{C}((\mathrm{Parity1}, 1, 0)) = 0 \quad \vee \quad \mathrm{Parity2} = 0$

42: $\mathrm{Parity}\underline{0} = 1$ by

39: $\mathrm{Parity1} = 1$

32: $\mathrm{Parity}\underline{0} = 1 \quad \vee \quad \neg\ \mathrm{Parity1} = 1$

43: $\neg\ \mathrm{Parity2} = 0 \quad \vee \quad \neg\ \mathrm{Parity}\underline{0} = 1 \quad \vee \quad \neg\ \mathrm{Parity}\underline{1} = 0$ by

39: $\mathrm{Parity1} = 1$

38: $\neg\ \mathrm{Parity1} = 1 \quad \vee \quad \neg\ \mathrm{Parity2} = 0 \quad \vee \quad \neg\ \mathrm{Parity}\underline{0} = 1 \quad \vee \quad \neg\ \mathrm{Parity}\underline{1} = 0$

44: $\mathrm{C}((\mathrm{Parity1}, 1, 0)) = 0$ by

39: $\mathrm{Parity1} = 1$

40: $\neg\ \mathrm{Parity1} = 1 \quad \vee \quad \mathrm{C}((\mathrm{Parity1}, 1, 0)) = 0$

45: $\neg\ \mathrm{Parity2} = 0 \quad \vee \quad \neg\ \mathrm{Parity}\underline{1} = 0$ by

42: $\mathrm{Parity}\underline{0} = 1$

43: $\neg\ \mathrm{Parity2} = 0 \quad \vee \quad \neg\ \mathrm{Parity}\underline{0} = 1 \quad \vee \quad \neg\ \mathrm{Parity}\underline{1} = 0$

46: $\mathrm{Parity2} = 0$ by

44: $\mathrm{C}((\mathrm{Parity1}, 1, 0)) = 0$

41: $\neg\ \mathrm{C}((\mathrm{Parity1}, 1, 0)) = 0 \quad \vee \quad \mathrm{Parity2} = 0$

47: $\mathrm{Parity}\underline{1} = 0$ by

46: $\mathrm{Parity2} = 0$

33: $\mathrm{Parity}\underline{1} = 0 \quad \vee \quad \neg\ \mathrm{Parity2} = 0$

48: $\neg\ \mathrm{Parity}\underline{1} = 0$ by

46: $\mathrm{Parity2} = 0$

45: $\neg\ \mathrm{Parity2} = 0 \quad \vee \quad \neg\ \mathrm{Parity}\underline{1} = 0$

49: QEA by

47: $\mathrm{Parity}\underline{1} = 0$

48: $\neg\ \mathrm{Parity}\underline{1} = 0$