Proof of Theorem 207

The theorem to be proved is

$\text{Parity}\, x = 1 \quad \rightarrow \quad \text{Parity}\, \text{S}x = 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(\text{Parity}x) = (1)] \quad \& \quad [\neg\, (\text{Parity}(\text{S}x)) = (0)]]$

Special cases of the hypothesis and previous results:

0: $\text{Parity}x = 1$ from H:x

1: $\neg\, \text{Parity}(\text{S}x) = 0$ from H:x

2: $\text{S}0 = 1$ from 115

3: $\text{C}((\text{Parity}x, 1, 0)) = \text{Parity}(\text{S}x)$ from 205;x

4: $\text{C}((\text{S}0, 1, 0)) = 0$ from 33;1;0;0

Equality substitutions:

5: $\neg\, \text{Parity}x = 1 \quad \vee \quad \neg\, \text{C}((\text{Parity}x, 1, 0)) = \text{Parity}(\text{S}x) \quad \vee \quad \text{C}((1, 1, 0)) = \text{Parity}(\text{S}x)$

6: $\neg\, \text{S}0 = 1 \quad \vee \quad \neg\, \text{C}((\text{S}0, 1, 0)) = 0 \quad \vee \quad \text{C}((1, 1, 0)) = 0$

7: $\neg\, \text{C}((1, 1, 0)) = \text{Parity}(\text{S}x) \quad \vee \quad \neg\, \text{C}((1, 1, 0)) = 0 \quad \vee \quad \text{Parity}(\text{S}x) = 0$

Inferences:

8: $\neg\, \text{C}((\text{Parity}x, 1, 0)) = \text{Parity}(\text{S}x) \quad \vee \quad \text{C}((1, 1, 0)) = \text{Parity}(\text{S}x)$ by
0: $\text{Parity}x = 1$
5: $\neg\, \text{Parity}x = 1 \quad \vee \quad \neg\, \text{C}((\text{Parity}x, 1, 0)) = \text{Parity}(\text{S}x) \quad \vee \quad \text{C}((1, 1, 0)) = \text{Parity}(\text{S}x)$

9: $\neg\, \text{C}((1, 1, 0)) = \text{Parity}(\text{S}x) \quad \vee \quad \neg\, \text{C}((1, 1, 0)) = 0$ by
1: $\neg\, \text{Parity}(\text{S}x) = 0$
7: $\neg\, \text{C}((1, 1, 0)) = \text{Parity}(\text{S}x) \quad \vee \quad \neg\, \text{C}((1, 1, 0)) = 0 \quad \vee \quad \text{Parity}(\text{S}x) = 0$

10: $\neg\, \text{C}((\text{S}0, 1, 0)) = 0 \quad \vee \quad \text{C}((1, 1, 0)) = 0$ by
2: $\text{S}0 = 1$
6: $\neg\, \text{S}0 = 1 \quad \vee \quad \neg\, \text{C}((\text{S}0, 1, 0)) = 0 \quad \vee \quad \text{C}((1, 1, 0)) = 0$

11: $\mathrm{C}((1,1,0)) = \mathrm{Parity}(\mathrm{S}x)$ by

3: $\mathrm{C}((\mathrm{Parity}x, 1, 0)) = \mathrm{Parity}(\mathrm{S}x)$

8: $\neg\, \mathrm{C}((\mathrm{Parity}x, 1, 0)) = \mathrm{Parity}(\mathrm{S}x) \quad \vee \quad \mathrm{C}((1,1,0)) = \mathrm{Parity}(\mathrm{S}x)$

12: $\mathrm{C}((1,1,0)) = 0$ by

4: $\mathrm{C}((\mathrm{S}0, 1, 0)) = 0$

10: $\neg\, \mathrm{C}((\mathrm{S}0, 1, 0)) = 0 \quad \vee \quad \mathrm{C}((1,1,0)) = 0$

13: $\neg\, \mathrm{C}((1,1,0)) = 0$ by

11: $\mathrm{C}((1,1,0)) = \mathrm{Parity}(\mathrm{S}x)$

9: $\neg\, \mathrm{C}((1,1,0)) = \mathrm{Parity}(\mathrm{S}x) \quad \vee \quad \neg\, \mathrm{C}((1,1,0)) = 0$

14: QEA by

12: $\mathrm{C}((1,1,0)) = 0$

13: $\neg\, \mathrm{C}((1,1,0)) = 0$