Proof of Theorem 206

The theorem to be proved is

$\text{Parity}\, x = 0 \quad \rightarrow \quad \text{Parity}\, \text{S}x = 1$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(\text{Parity}x) = (0)] \quad \& \quad [\neg\, (\text{Parity}(\text{S}x)) = (1)]]$

Special cases of the hypothesis and previous results:

0: $\text{Parity}x = 0$ from H:x

1: $\neg\, \text{Parity}(\text{S}x) = 1$ from H:x

2: $\text{C}((\text{Parity}x, 1, 0)) = \text{Parity}(\text{S}x)$ from 205;x

3: $\text{C}((0, 1, 0)) = 1$ from 33;1;0;0

Equality substitutions:

4: $\neg\, \text{Parity}x = 0 \quad \vee \quad \neg\, \text{C}((\text{Parity}x, 1, 0)) = \text{Parity}(\text{S}x) \quad \vee \quad \text{C}((0, 1, 0)) = \text{Parity}(\text{S}x)$

5: $\neg\, \text{C}((0, 1, 0)) = 1 \quad \vee \quad \neg\, \text{C}((0, 1, 0)) = \text{Parity}(\text{S}x) \quad \vee \quad 1 = \text{Parity}(\text{S}x)$

Inferences:

6: $\neg\, \text{C}((\text{Parity}x, 1, 0)) = \text{Parity}(\text{S}x) \quad \vee \quad \text{C}((0, 1, 0)) = \text{Parity}(\text{S}x)$ by
0: $\text{Parity}x = 0$
4: $\neg\, \text{Parity}x = 0 \quad \vee \quad \neg\, \text{C}((\text{Parity}x, 1, 0)) = \text{Parity}(\text{S}x) \quad \vee \quad \text{C}((0, 1, 0)) = \text{Parity}(\text{S}x)$

7: $\neg\, \text{C}((0, 1, 0)) = 1 \quad \vee \quad \neg\, \text{C}((0, 1, 0)) = \text{Parity}(\text{S}x)$ by
1: $\neg\, \text{Parity}(\text{S}x) = 1$
5: $\neg\, \text{C}((0, 1, 0)) = 1 \quad \vee \quad \neg\, \text{C}((0, 1, 0)) = \text{Parity}(\text{S}x) \quad \vee \quad \text{Parity}(\text{S}x) = 1$

8: $\text{C}((0, 1, 0)) = \text{Parity}(\text{S}x)$ by
2: $\text{C}((\text{Parity}x, 1, 0)) = \text{Parity}(\text{S}x)$
6: $\neg\, \text{C}((\text{Parity}x, 1, 0)) = \text{Parity}(\text{S}x) \quad \vee \quad \text{C}((0, 1, 0)) = \text{Parity}(\text{S}x)$

9: $\neg\, \text{C}((0, 1, 0)) = \text{Parity}(\text{S}x)$ by
3: $\text{C}((0, 1, 0)) = 1$
7: $\neg\, \text{C}((0, 1, 0)) = 1 \quad \vee \quad \neg\, \text{C}((0, 1, 0)) = \text{Parity}(\text{S}x)$

10: QEA by
8: $\text{C}((0, 1, 0)) = \text{Parity}(\text{S}x)$
9: $\neg\, \text{C}((0, 1, 0)) = \text{Parity}(\text{S}x)$