Proof of Theorem 203

The theorem to be proved is

$Qx = 1 \quad \rightarrow \quad x = \epsilon$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(Qx) = (1)] \quad \& \quad [\neg\, (x) = (\epsilon)]]$

Special cases of the hypothesis and previous results:

0: $Qx = 1$ from H:x

1: $\neg\, \epsilon = x$ from H:x

2: $S0 = 1$ from 115

3: $\epsilon = 0$ from 185

4: $1 + 0 = 1$ from 12;1

5: $(Qx) + (Rx) = Sx$ from 166;x

6: $Rx < Qx$ from 166;x

7: $\neg\, Rx \leq 1 \quad \vee \quad Rx = 0 \quad \vee \quad Rx = 1$ from 200;Rx

8: $\neg\, Rx < 1 \quad \vee \quad Rx \leq 1$ from $56^{\rightarrow}$;Rx;1

9: $\neg\, Rx < 1 \quad \vee \quad \neg\, Rx = 1$ from $56^{\rightarrow}$;Rx;1

10: $\neg\, S0 = Sx \quad \vee \quad 0 = x$ from 4;x;0

Equality substitutions:

11: $\neg\, Qx = 1 \quad \vee \quad \neg\, (Qx) + (Rx) = Sx \quad \vee \quad (1) + (Rx) = Sx$

12: $\neg\, Qx = 1 \quad \vee \quad \neg\, Rx < Qx \quad \vee \quad Rx < 1$

13: $\neg\, S0 = 1 \quad \vee \quad S0 = Sx \quad \vee \quad \neg\, 1 = Sx$

14: $\neg\, \epsilon = 0 \quad \vee \quad \epsilon = x \quad \vee \quad \neg\, 0 = x$

15: $\neg\, 1 + 0 = 1 \quad \vee \quad \neg\, 1 + 0 = Sx \quad \vee \quad 1 = Sx$

16: $\neg\, Rx = 0 \quad \vee \quad \neg\, 1 + (Rx) = Sx \quad \vee \quad 1 + (0) = Sx$

Inferences:

17: $\neg\,(\mathrm{Q}x) + (\mathrm{R}x) = \mathrm{S}x \quad \vee \quad 1 + (\mathrm{R}x) = \mathrm{S}x$ by

0: $\mathrm{Q}x = 1$

11: $\neg\,\mathrm{Q}x = 1 \quad \vee \quad \neg\,(\mathrm{Q}x) + (\mathrm{R}x) = \mathrm{S}x \quad \vee \quad 1 + (\mathrm{R}x) = \mathrm{S}x$

18: $\neg\,\mathrm{R}x < \mathrm{Q}x \quad \vee \quad \mathrm{R}x < 1$ by

0: $\mathrm{Q}x = 1$

12: $\neg\,\mathrm{Q}x = 1 \quad \vee \quad \neg\,\mathrm{R}x < \mathrm{Q}x \quad \vee \quad \mathrm{R}x < 1$

19: $\neg\,\epsilon = 0 \quad \vee \quad \neg\,0 = x$ by

1: $\neg\,\epsilon = x$

14: $\neg\,\epsilon = 0 \quad \vee \quad \epsilon = x \quad \vee \quad \neg\,0 = x$

20: $\mathrm{S}0 = \mathrm{S}x \quad \vee \quad \neg\,\mathrm{S}x = 1$ by

2: $\mathrm{S}0 = 1$

13: $\neg\,\mathrm{S}0 = 1 \quad \vee \quad \mathrm{S}0 = \mathrm{S}x \quad \vee \quad \neg\,\mathrm{S}x = 1$

21: $\neg\,0 = x$ by

3: $\epsilon = 0$

19: $\neg\,\epsilon = 0 \quad \vee \quad \neg\,0 = x$

22: $\neg\,1 + 0 = \mathrm{S}x \quad \vee \quad \mathrm{S}x = 1$ by

4: $1 + 0 = 1$

15: $\neg\,1 + 0 = 1 \quad \vee \quad \neg\,1 + 0 = \mathrm{S}x \quad \vee \quad \mathrm{S}x = 1$

23: $1 + (\mathrm{R}x) = \mathrm{S}x$ by

5: $(\mathrm{Q}x) + (\mathrm{R}x) = \mathrm{S}x$

17: $\neg\,(\mathrm{Q}x) + (\mathrm{R}x) = \mathrm{S}x \quad \vee \quad 1 + (\mathrm{R}x) = \mathrm{S}x$

24: $\mathrm{R}x < 1$ by

6: $\mathrm{R}x < \mathrm{Q}x$

18: $\neg\,\mathrm{R}x < \mathrm{Q}x \quad \vee \quad \mathrm{R}x < 1$

25: $\neg\,\mathrm{S}0 = \mathrm{S}x$ by

21: $\neg\,0 = x$

10: $\neg\,\mathrm{S}0 = \mathrm{S}x \quad \vee \quad 0 = x$

26: $\neg\,\mathrm{R}x = 0 \quad \vee \quad 1 + 0 = \mathrm{S}x$ by

23: $1 + (\mathrm{R}x) = \mathrm{S}x$

16: $\neg\,\mathrm{R}x = 0 \quad \vee \quad \neg\,1 + (\mathrm{R}x) = \mathrm{S}x \quad \vee \quad 1 + 0 = \mathrm{S}x$

27: $\mathrm{R}x \leq 1$ by

24: $\mathrm{R}x < 1$

8: $\neg\,\mathrm{R}x < 1 \quad \vee \quad \mathrm{R}x \leq 1$

28: $\neg \, \mathrm{R}x = 1$ by

24: $\mathrm{R}x < 1$

9: $\neg \, \mathrm{R}x < 1 \quad \vee \quad \neg \, \mathrm{R}x = 1$

29: $\neg \, \mathrm{S}x = 1$ by

25: $\neg \, \mathrm{S}0 = \mathrm{S}x$

20: $\mathrm{S}0 = \mathrm{S}x \quad \vee \quad \neg \, \mathrm{S}x = 1$

30: $\mathrm{R}x = 0 \quad \vee \quad \mathrm{R}x = 1$ by

27: $\mathrm{R}x \leq 1$

7: $\neg \, \mathrm{R}x \leq 1 \quad \vee \quad \mathrm{R}x = 0 \quad \vee \quad \mathrm{R}x = 1$

31: $\mathrm{R}x = 0$ by

28: $\neg \, \mathrm{R}x = 1$

30: $\mathrm{R}x = 0 \quad \vee \quad \mathrm{R}x = 1$

32: $\neg \, 1 + 0 = \mathrm{S}x$ by

29: $\neg \, \mathrm{S}x = 1$

22: $\neg \, 1 + 0 = \mathrm{S}x \quad \vee \quad \mathrm{S}x = 1$

33: $1 + 0 = \mathrm{S}x$ by

31: $\mathrm{R}x = 0$

26: $\neg \, \mathrm{R}x = 0 \quad \vee \quad 1 + 0 = \mathrm{S}x$

34: QEA by

32: $\neg \, 1 + 0 = \mathrm{S}x$

33: $1 + 0 = \mathrm{S}x$