Proof of Theorem 19i

The theorem to be proved is

$x - x = 0 \quad \rightarrow \quad \mathrm{S}x - \mathrm{S}x = 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x - x) = (0)] \quad \& \quad [\neg\, ((\mathrm{S}x) - (\mathrm{S}x)) = (0)]]$

Special cases of the hypothesis and previous results:

0: $x - x = 0$ from H:x

1: $\neg\, (\mathrm{S}x) - (\mathrm{S}x) = 0$ from H:x

2: $(\mathrm{S}x) - (\mathrm{S}x) = x - x$ from 18;x;x

Equality substitutions:

3: $\neg\, x - x = 0 \quad \vee \quad \neg\, (\mathrm{S}x) - (\mathrm{S}x) = x - x \quad \vee \quad (\mathrm{S}x) - (\mathrm{S}x) = 0$

Inferences:

4: $\neg\, (\mathrm{S}x) - (\mathrm{S}x) = x - x \quad \vee \quad (\mathrm{S}x) - (\mathrm{S}x) = 0$ by
0: $x - x = 0$
3: $\neg\, x - x = 0 \quad \vee \quad \neg\, (\mathrm{S}x) - (\mathrm{S}x) = x - x \quad \vee \quad (\mathrm{S}x) - (\mathrm{S}x) = 0$

5: $\neg\, (\mathrm{S}x) - (\mathrm{S}x) = x - x$ by
1: $\neg\, (\mathrm{S}x) - (\mathrm{S}x) = 0$
4: $\neg\, (\mathrm{S}x) - (\mathrm{S}x) = x - x \quad \vee \quad (\mathrm{S}x) - (\mathrm{S}x) = 0$

6: QEA by
2: $(\mathrm{S}x) - (\mathrm{S}x) = x - x$
5: $\neg\, (\mathrm{S}x) - (\mathrm{S}x) = x - x$