Proof of Theorem 199

The theorem to be proved is

$x \oplus y = x \oplus z \quad \rightarrow \quad y = z$ ⋆

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x \oplus y) = (x \oplus z)] \quad \& \quad [\neg\ (y) = (z)]]$

Special cases of the hypothesis and previous results:

0: $x \oplus z = x \oplus y$ from H:x:y:z

1: $\neg\ z = y$ from H:x:y:z

2: $(\mathrm{Q}x) \cdot (\mathrm{Q}y) = \mathrm{Q}(x \oplus y)$ from 180;x;y

3: $((\mathrm{R}x) \cdot (\mathrm{Q}y)) + (\mathrm{R}y) = \mathrm{R}(x \oplus y)$ from 180;x;y

4: $(\mathrm{Q}x) \cdot (\mathrm{Q}z) = \mathrm{Q}(x \oplus z)$ from 180;x;z

5: $((\mathrm{R}x) \cdot (\mathrm{Q}z)) + (\mathrm{R}z) = \mathrm{R}(x \oplus z)$ from 180;x;z

6: $\neg\ \mathrm{Q}x = 0$ from 178;x

7: $\neg\ (\mathrm{Q}x) \cdot (\mathrm{Q}z) = (\mathrm{Q}x) \cdot (\mathrm{Q}y) \quad \vee \quad \mathrm{Q}x = 0 \quad \vee \quad \mathrm{Q}z = \mathrm{Q}y$ from 198;Qx;Qy;Qz

8: $\neg\ ((\mathrm{R}x) \cdot (\mathrm{Q}y)) + (\mathrm{R}z) = ((\mathrm{R}x) \cdot (\mathrm{Q}y)) + (\mathrm{R}y) \quad \vee \quad \mathrm{R}z = \mathrm{R}y$ from 120;Rx) $\cdot$ (Qy;(Ry);Rz

9: $\neg\ \mathrm{Q}z = \mathrm{Q}y \quad \vee \quad \neg\ \mathrm{R}z = \mathrm{R}y \quad \vee \quad z = y$ from 193;y;z

Equality substitutions:

10: $\neg\ x \oplus z = x \oplus y \quad \vee \quad (\mathrm{Q}x) \cdot (\mathrm{Q}y) = \mathrm{Q}(x \oplus z) \quad \vee \quad \neg\ (\mathrm{Q}x) \cdot (\mathrm{Q}y) = \mathrm{Q}(x \oplus y)$

11: $\neg\ x \oplus z = x \oplus y \quad \vee \quad ((\mathrm{R}x) \cdot (\mathrm{Q}y)) + (\mathrm{R}y) = \mathrm{R}(x \oplus z) \quad \vee \quad \neg\ ((\mathrm{R}x) \cdot (\mathrm{Q}y)) + (\mathrm{R}y) = \mathrm{R}(x \oplus y)$

12: $\neg\ (\mathrm{Q}x) \cdot (\mathrm{Q}z) = \mathrm{Q}(x \oplus z) \quad \vee \quad (\mathrm{Q}x) \cdot (\mathrm{Q}z) = (\mathrm{Q}x) \cdot (\mathrm{Q}y) \quad \vee \quad \neg\ \mathrm{Q}(x \oplus z) = (\mathrm{Q}x) \cdot (\mathrm{Q}y)$

13: $\neg\ \mathrm{Q}z = \mathrm{Q}y \quad \vee \quad \neg\ ((\mathrm{R}x) \cdot (\mathrm{Q}z)) + (\mathrm{R}z) = \mathrm{R}(x \oplus z) \quad \vee \quad ((\mathrm{R}x) \cdot (\mathrm{Q}y)) + (\mathrm{R}z) = \mathrm{R}(x \oplus z)$

14: $\neg\ ((\mathrm{R}x) \cdot (\mathrm{Q}y)) + (\mathrm{R}z) = \mathrm{R}(x \oplus z) \quad \vee \quad ((\mathrm{R}x) \cdot (\mathrm{Q}y)) + (\mathrm{R}z) = ((\mathrm{R}x) \cdot (\mathrm{Q}y)) + (\mathrm{R}y) \quad \vee \quad \neg\ \mathrm{R}(x \oplus z) = ((\mathrm{R}x) \cdot (\mathrm{Q}y)) + (\mathrm{R}y)$

Inferences:

15: $(\mathrm{Q}x)\cdot(\mathrm{Q}y) = \mathrm{Q}(x \oplus z) \quad \vee \quad \neg\,(\mathrm{Q}x)\cdot(\mathrm{Q}y) = \mathrm{Q}(x \oplus y)$ by

0: $x \oplus z = x \oplus y$

10: $\neg\, x \oplus z = x \oplus y \quad \vee \quad (\mathrm{Q}x)\cdot(\mathrm{Q}y) = \mathrm{Q}(x \oplus z) \quad \vee \quad \neg\,(\mathrm{Q}x)\cdot(\mathrm{Q}y) = \mathrm{Q}(x \oplus y)$

16: $((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y) = \mathrm{R}(x \oplus z) \quad \vee \quad \neg\,((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y) = \mathrm{R}(x \oplus y)$ by

0: $x \oplus z = x \oplus y$

11: $\neg\, x \oplus z = x \oplus y \quad \vee \quad ((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y) = \mathrm{R}(x \oplus z) \quad \vee \quad \neg\,((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y) = \mathrm{R}(x \oplus y)$

17: $\neg\, \mathrm{Q}z = \mathrm{Q}y \quad \vee \quad \neg\, \mathrm{R}z = \mathrm{R}y$ by

1: $\neg\, z = y$

9: $\neg\, \mathrm{Q}z = \mathrm{Q}y \quad \vee \quad \neg\, \mathrm{R}z = \mathrm{R}y \quad \vee \quad z = y$

18: $(\mathrm{Q}x)\cdot(\mathrm{Q}y) = \mathrm{Q}(x \oplus z)$ by

2: $(\mathrm{Q}x)\cdot(\mathrm{Q}y) = \mathrm{Q}(x \oplus y)$

15: $(\mathrm{Q}x)\cdot(\mathrm{Q}y) = \mathrm{Q}(x \oplus z) \quad \vee \quad \neg\,(\mathrm{Q}x)\cdot(\mathrm{Q}y) = \mathrm{Q}(x \oplus y)$

19: $((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y) = \mathrm{R}(x \oplus z)$ by

3: $((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y) = \mathrm{R}(x \oplus y)$

16: $((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y) = \mathrm{R}(x \oplus z) \quad \vee \quad \neg\,((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y) = \mathrm{R}(x \oplus y)$

20: $(\mathrm{Q}x)\cdot(\mathrm{Q}z) = (\mathrm{Q}x)\cdot(\mathrm{Q}y) \quad \vee \quad \neg\,(\mathrm{Q}x)\cdot(\mathrm{Q}y) = \mathrm{Q}(x \oplus z)$ by

4: $(\mathrm{Q}x)\cdot(\mathrm{Q}z) = \mathrm{Q}(x \oplus z)$

12: $\neg\,(\mathrm{Q}x)\cdot(\mathrm{Q}z) = \mathrm{Q}(x \oplus z) \quad \vee \quad (\mathrm{Q}x)\cdot(\mathrm{Q}z) = (\mathrm{Q}x)\cdot(\mathrm{Q}y) \quad \vee \quad \neg\,(\mathrm{Q}x)\cdot(\mathrm{Q}y) = \mathrm{Q}(x \oplus z)$

21: $\neg\, \mathrm{Q}z = \mathrm{Q}y \quad \vee \quad ((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}z) = \mathrm{R}(x \oplus z)$ by

5: $((\mathrm{R}x)\cdot(\mathrm{Q}z)) + (\mathrm{R}z) = \mathrm{R}(x \oplus z)$

13: $\neg\, \mathrm{Q}z = \mathrm{Q}y \quad \vee \quad \neg\,((\mathrm{R}x)\cdot(\mathrm{Q}z)) + (\mathrm{R}z) = \mathrm{R}(x \oplus z) \quad \vee \quad ((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}z) = \mathrm{R}(x \oplus z)$

22: $\neg\,(\mathrm{Q}x)\cdot(\mathrm{Q}z) = (\mathrm{Q}x)\cdot(\mathrm{Q}y) \quad \vee \quad \mathrm{Q}z = \mathrm{Q}y$ by

6: $\neg\, \mathrm{Q}x = 0$

7: $\neg\,(\mathrm{Q}x)\cdot(\mathrm{Q}z) = (\mathrm{Q}x)\cdot(\mathrm{Q}y) \quad \vee \quad \mathrm{Q}x = 0 \quad \vee \quad \mathrm{Q}z = \mathrm{Q}y$

23: $(\mathrm{Q}x)\cdot(\mathrm{Q}z) = (\mathrm{Q}x)\cdot(\mathrm{Q}y)$ by

18: $(\mathrm{Q}x)\cdot(\mathrm{Q}y) = \mathrm{Q}(x \oplus z)$

20: $(\mathrm{Q}x)\cdot(\mathrm{Q}z) = (\mathrm{Q}x)\cdot(\mathrm{Q}y) \quad \vee \quad \neg\,(\mathrm{Q}x)\cdot(\mathrm{Q}y) = \mathrm{Q}(x \oplus z)$

24: $\neg\,((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}z) = \mathrm{R}(x \oplus z) \quad \vee \quad ((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}z) = ((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y)$ by

19: $((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y) = \mathrm{R}(x \oplus z)$

14: $\neg\,((\mathrm{R}x)\cdot(\mathrm{Q}y))+(\mathrm{R}z)=\mathrm{R}(x\oplus z)\quad\vee\quad((\mathrm{R}x)\cdot(\mathrm{Q}y))+(\mathrm{R}z)=((\mathrm{R}x)\cdot(\mathrm{Q}y))+(\mathrm{R}y)$
$\vee\quad\neg\,((\mathrm{R}x)\cdot(\mathrm{Q}y))+(\mathrm{R}y)=\mathrm{R}(x\oplus z)$

25: $\mathrm{Q}z=\mathrm{Q}y$ by
23: $(\mathrm{Q}x)\cdot(\mathrm{Q}z)=(\mathrm{Q}x)\cdot(\mathrm{Q}y)$
22: $\neg\,(\mathrm{Q}x)\cdot(\mathrm{Q}z)=(\mathrm{Q}x)\cdot(\mathrm{Q}y)\quad\vee\quad\mathrm{Q}z=\mathrm{Q}y$

26: $\neg\,\mathrm{R}z=\mathrm{R}y$ by
25: $\mathrm{Q}z=\mathrm{Q}y$
17: $\neg\,\mathrm{Q}z=\mathrm{Q}y\quad\vee\quad\neg\,\mathrm{R}z=\mathrm{R}y$

27: $((\mathrm{R}x)\cdot(\mathrm{Q}y))+(\mathrm{R}z)=\mathrm{R}(x\oplus z)$ by
25: $\mathrm{Q}z=\mathrm{Q}y$
21: $\neg\,\mathrm{Q}z=\mathrm{Q}y\quad\vee\quad((\mathrm{R}x)\cdot(\mathrm{Q}y))+(\mathrm{R}z)=\mathrm{R}(x\oplus z)$

28: $\neg\,((\mathrm{R}x)\cdot(\mathrm{Q}y))+(\mathrm{R}z)=((\mathrm{R}x)\cdot(\mathrm{Q}y))+(\mathrm{R}y)$ by
26: $\neg\,\mathrm{R}z=\mathrm{R}y$
8: $\neg\,((\mathrm{R}x)\cdot(\mathrm{Q}y))+(\mathrm{R}z)=((\mathrm{R}x)\cdot(\mathrm{Q}y))+(\mathrm{R}y)\quad\vee\quad\mathrm{R}z=\mathrm{R}y$

29: $((\mathrm{R}x)\cdot(\mathrm{Q}y))+(\mathrm{R}z)=((\mathrm{R}x)\cdot(\mathrm{Q}y))+(\mathrm{R}y)$ by
27: $((\mathrm{R}x)\cdot(\mathrm{Q}y))+(\mathrm{R}z)=\mathrm{R}(x\oplus z)$
24: $\neg\,((\mathrm{R}x)\cdot(\mathrm{Q}y))+(\mathrm{R}z)=\mathrm{R}(x\oplus z)\quad\vee\quad((\mathrm{R}x)\cdot(\mathrm{Q}y))+(\mathrm{R}z)=((\mathrm{R}x)\cdot(\mathrm{Q}y))+(\mathrm{R}y)$

30: QEA by
28: $\neg\,((\mathrm{R}x)\cdot(\mathrm{Q}y))+(\mathrm{R}z)=((\mathrm{R}x)\cdot(\mathrm{Q}y))+(\mathrm{R}y)$
29: $((\mathrm{R}x)\cdot(\mathrm{Q}y))+(\mathrm{R}z)=((\mathrm{R}x)\cdot(\mathrm{Q}y))+(\mathrm{R}y)$