Proof of Theorem 198

The theorem to be proved is

$x \cdot y = x \cdot z \quad \& \quad x \neq 0 \quad \rightarrow \quad y = z$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x \cdot y) = (x \cdot z)] \quad \& \quad [\neg\,(x) = (0)] \quad \& \quad [\neg\,(y) = (z)]]$

Special cases of the hypothesis and previous results:

0: $x \cdot z = x \cdot y$ from H:x:y:z

1: $\neg\, 0 = x$ from H:x:y:z

2: $\neg\, z = y$ from H:x:y:z

3: $y \cdot x = x \cdot y$ from 105;x;y

4: $z \cdot x = x \cdot z$ from 105;x;z

5: $\neg\, z \cdot x = y \cdot x \quad \vee \quad 0 = x \quad \vee \quad z = y$ from 141;y;x;z

Equality substitutions:

6: $\neg\, x \cdot z = x \cdot y \quad \vee \quad y \cdot x = x \cdot z \quad \vee \quad \neg\, y \cdot x = x \cdot y$

7: $\neg\, z \cdot x = x \cdot z \quad \vee \quad z \cdot x = y \cdot x \quad \vee \quad \neg\, x \cdot z = y \cdot x$

Inferences:

8: $y \cdot x = x \cdot z \quad \vee \quad \neg\, y \cdot x = x \cdot y$ by
0: $x \cdot z = x \cdot y$
6: $\neg\, x \cdot z = x \cdot y \quad \vee \quad y \cdot x = x \cdot z \quad \vee \quad \neg\, y \cdot x = x \cdot y$

9: $\neg\, z \cdot x = y \cdot x \quad \vee \quad z = y$ by
1: $\neg\, 0 = x$
5: $\neg\, z \cdot x = y \cdot x \quad \vee \quad 0 = x \quad \vee \quad z = y$

10: $\neg\, z \cdot x = y \cdot x$ by
2: $\neg\, z = y$
9: $\neg\, z \cdot x = y \cdot x \quad \vee \quad z = y$

11: $y \cdot x = x \cdot z$ by

3: $y \cdot x = x \cdot y$

8: $y \cdot x = x \cdot z \quad \vee \quad \neg\, y \cdot x = x \cdot y$

12: $z \cdot x = y \cdot x \quad \vee \quad \neg\, y \cdot x = x \cdot z$ by

4: $z \cdot x = x \cdot z$

7: $\neg\, z \cdot x = x \cdot z \quad \vee \quad z \cdot x = y \cdot x \quad \vee \quad \neg\, y \cdot x = x \cdot z$

13: $\neg\, y \cdot x = x \cdot z$ by

10: $\neg\, z \cdot x = y \cdot x$

12: $z \cdot x = y \cdot x \quad \vee \quad \neg\, y \cdot x = x \cdot z$

14: QEA by

11: $y \cdot x = x \cdot z$

13: $\neg\, y \cdot x = x \cdot z$