Proof of Theorem 196

The theorem to be proved is

$x \oplus \epsilon = x$ ⋆

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(x \oplus \epsilon) = (x)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, x \oplus \epsilon = x$ from H:x

1: $\neg\, \mathrm{Q}(x \oplus \epsilon) = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{R}(x \oplus \epsilon) = \mathrm{R}x \quad \vee \quad x \oplus \epsilon = x$ from 193;$x \oplus \epsilon$;x

2: $\mathrm{Q}\epsilon = 1$ from 189

3: $\mathrm{R}\epsilon = 0$ from 189

4: $(\mathrm{Q}x) \cdot (\mathrm{Q}\epsilon) = \mathrm{Q}(x \oplus \epsilon)$ from 180;x;ϵ

5: $((\mathrm{R}x) \cdot (\mathrm{Q}\epsilon)) + (\mathrm{R}\epsilon) = \mathrm{R}(x \oplus \epsilon)$ from 180;x;ϵ

6: $(\mathrm{Q}x) \cdot 1 = \mathrm{Q}x$ from 195;$\mathrm{Q}x$

7: $(\mathrm{R}x) \cdot 1 = \mathrm{R}x$ from 195;$\mathrm{R}x$

8: $(\mathrm{R}x) + 0 = \mathrm{R}x$ from 12;$\mathrm{R}x$

Equality substitutions:

9: $\neg\, \mathrm{Q}\epsilon = 1 \quad \vee \quad \neg\, (\mathrm{Q}x) \cdot (\mathrm{Q}\epsilon) = \mathrm{Q}(x \oplus \epsilon) \quad \vee \quad (\mathrm{Q}x) \cdot (1) = \mathrm{Q}(x \oplus \epsilon)$

10: $\neg\, \mathrm{Q}\epsilon = 1 \quad \vee \quad \neg\, ((\mathrm{R}x) \cdot (\mathrm{Q}\epsilon)) + (\mathrm{R}\epsilon) = \mathrm{R}(x \oplus \epsilon) \quad \vee \quad ((\mathrm{R}x) \cdot (1)) + (\mathrm{R}\epsilon) = \mathrm{R}(x \oplus \epsilon)$

11: $\neg\, \mathrm{R}\epsilon = 0 \quad \vee \quad \neg\, ((\mathrm{R}x) \cdot 1) + (\mathrm{R}\epsilon) = \mathrm{R}(x \oplus \epsilon) \quad \vee \quad ((\mathrm{R}x) \cdot 1) + (0) = \mathrm{R}(x \oplus \epsilon)$

12: $\neg\, (\mathrm{Q}x) \cdot 1 = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{Q}(x \oplus \epsilon) = (\mathrm{Q}x) \cdot 1 \quad \vee \quad \mathrm{Q}(x \oplus \epsilon) = \mathrm{Q}x$

13: $\neg\, (\mathrm{R}x) \cdot 1 = \mathrm{R}x \quad \vee \quad ((\mathrm{R}x) \cdot 1) + 0 = \mathrm{R}x \quad \vee \quad \neg\, (\mathrm{R}x) + 0 = \mathrm{R}x$

14: $\neg\, ((\mathrm{R}x) \cdot 1) + 0 = \mathrm{R}(x \oplus \epsilon) \quad \vee \quad \neg\, ((\mathrm{R}x) \cdot 1) + 0 = \mathrm{R}x \quad \vee \quad \mathrm{R}(x \oplus \epsilon) = \mathrm{R}x$

Inferences:

15: $\neg\, \mathrm{Q}(x \oplus \epsilon) = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{R}(x \oplus \epsilon) = \mathrm{R}x$ by

0: $\neg\, x \oplus \epsilon = x$

1: $\neg\, \mathrm{Q}(x \oplus \epsilon) = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{R}(x \oplus \epsilon) = \mathrm{R}x \quad \vee \quad x \oplus \epsilon = x$

16: $\neg\,(\mathrm{Q}x)\cdot(\mathrm{Q}\epsilon) = \mathrm{Q}(x \oplus \epsilon) \quad \vee \quad \mathrm{Q}(x \oplus \epsilon) = (\mathrm{Q}x)\cdot 1$ by

2: $\mathrm{Q}\epsilon = 1$

9: $\neg\,\mathrm{Q}\epsilon = 1 \quad \vee \quad \neg\,(\mathrm{Q}x)\cdot(\mathrm{Q}\epsilon) = \mathrm{Q}(x \oplus \epsilon) \quad \vee \quad \mathrm{Q}(x \oplus \epsilon) = (\mathrm{Q}x)\cdot 1$

17: $\neg\,((\mathrm{R}x)\cdot(\mathrm{Q}\epsilon)) + (\mathrm{R}\epsilon) = \mathrm{R}(x \oplus \epsilon) \quad \vee \quad ((\mathrm{R}x)\cdot 1) + (\mathrm{R}\epsilon) = \mathrm{R}(x \oplus \epsilon)$ by

2: $\mathrm{Q}\epsilon = 1$

10: $\neg\,\mathrm{Q}\epsilon = 1 \quad \vee \quad \neg\,((\mathrm{R}x)\cdot(\mathrm{Q}\epsilon)) + (\mathrm{R}\epsilon) = \mathrm{R}(x \oplus \epsilon) \quad \vee \quad ((\mathrm{R}x)\cdot 1) + (\mathrm{R}\epsilon) = \mathrm{R}(x \oplus \epsilon)$

18: $\neg\,((\mathrm{R}x)\cdot 1) + (\mathrm{R}\epsilon) = \mathrm{R}(x \oplus \epsilon) \quad \vee \quad ((\mathrm{R}x)\cdot 1) + 0 = \mathrm{R}(x \oplus \epsilon)$ by

3: $\mathrm{R}\epsilon = 0$

11: $\neg\,\mathrm{R}\epsilon = 0 \quad \vee \quad \neg\,((\mathrm{R}x)\cdot 1) + (\mathrm{R}\epsilon) = \mathrm{R}(x \oplus \epsilon) \quad \vee \quad ((\mathrm{R}x)\cdot 1) + 0 = \mathrm{R}(x \oplus \epsilon)$

19: $\mathrm{Q}(x \oplus \epsilon) = (\mathrm{Q}x)\cdot 1$ by

4: $(\mathrm{Q}x)\cdot(\mathrm{Q}\epsilon) = \mathrm{Q}(x \oplus \epsilon)$

16: $\neg\,(\mathrm{Q}x)\cdot(\mathrm{Q}\epsilon) = \mathrm{Q}(x \oplus \epsilon) \quad \vee \quad \mathrm{Q}(x \oplus \epsilon) = (\mathrm{Q}x)\cdot 1$

20: $((\mathrm{R}x)\cdot 1) + (\mathrm{R}\epsilon) = \mathrm{R}(x \oplus \epsilon)$ by

5: $((\mathrm{R}x)\cdot(\mathrm{Q}\epsilon)) + (\mathrm{R}\epsilon) = \mathrm{R}(x \oplus \epsilon)$

17: $\neg\,((\mathrm{R}x)\cdot(\mathrm{Q}\epsilon)) + (\mathrm{R}\epsilon) = \mathrm{R}(x \oplus \epsilon) \quad \vee \quad ((\mathrm{R}x)\cdot 1) + (\mathrm{R}\epsilon) = \mathrm{R}(x \oplus \epsilon)$

21: $\neg\,\mathrm{Q}(x \oplus \epsilon) = (\mathrm{Q}x)\cdot 1 \quad \vee \quad \mathrm{Q}(x \oplus \epsilon) = \mathrm{Q}x$ by

6: $(\mathrm{Q}x)\cdot 1 = \mathrm{Q}x$

12: $\neg\,(\mathrm{Q}x)\cdot 1 = \mathrm{Q}x \quad \vee \quad \neg\,\mathrm{Q}(x \oplus \epsilon) = (\mathrm{Q}x)\cdot 1 \quad \vee \quad \mathrm{Q}(x \oplus \epsilon) = \mathrm{Q}x$

22: $((\mathrm{R}x)\cdot 1) + 0 = \mathrm{R}x \quad \vee \quad \neg\,(\mathrm{R}x) + 0 = \mathrm{R}x$ by

7: $(\mathrm{R}x)\cdot 1 = \mathrm{R}x$

13: $\neg\,(\mathrm{R}x)\cdot 1 = \mathrm{R}x \quad \vee \quad ((\mathrm{R}x)\cdot 1) + 0 = \mathrm{R}x \quad \vee \quad \neg\,(\mathrm{R}x) + 0 = \mathrm{R}x$

23: $((\mathrm{R}x)\cdot 1) + 0 = \mathrm{R}x$ by

8: $(\mathrm{R}x) + 0 = \mathrm{R}x$

22: $((\mathrm{R}x)\cdot 1) + 0 = \mathrm{R}x \quad \vee \quad \neg\,(\mathrm{R}x) + 0 = \mathrm{R}x$

24: $\mathrm{Q}(x \oplus \epsilon) = \mathrm{Q}x$ by

19: $\mathrm{Q}(x \oplus \epsilon) = (\mathrm{Q}x)\cdot 1$

21: $\neg\,\mathrm{Q}(x \oplus \epsilon) = (\mathrm{Q}x)\cdot 1 \quad \vee \quad \mathrm{Q}(x \oplus \epsilon) = \mathrm{Q}x$

25: $((\mathrm{R}x)\cdot 1) + 0 = \mathrm{R}(x \oplus \epsilon)$ by

20: $((\mathrm{R}x)\cdot 1) + (\mathrm{R}\epsilon) = \mathrm{R}(x \oplus \epsilon)$

18: $\neg\,((\mathrm{R}x)\cdot 1) + (\mathrm{R}\epsilon) = \mathrm{R}(x \oplus \epsilon) \quad \vee \quad ((\mathrm{R}x)\cdot 1) + 0 = \mathrm{R}(x \oplus \epsilon)$

26: $\neg\,((\mathrm{R}x)\cdot 1) + 0 = \mathrm{R}(x \oplus \epsilon) \quad \vee \quad \mathrm{R}(x \oplus \epsilon) = \mathrm{R}x$ by

23: $((\mathrm{R}x)\cdot 1) + 0 = \mathrm{R}x$

14: $\neg\,((\mathrm{R}x)\cdot 1) + 0 = \mathrm{R}(x \oplus \epsilon) \quad \vee \quad \neg\,((\mathrm{R}x)\cdot 1) + 0 = \mathrm{R}x \quad \vee \quad \mathrm{R}(x \oplus \epsilon) = \mathrm{R}x$

27: $\neg\ \mathrm{R}(x \oplus \epsilon) = \mathrm{R}x$ by

24: $\mathrm{Q}(x \oplus \epsilon) = \mathrm{Q}x$

15: $\neg\ \mathrm{Q}(x \oplus \epsilon) = \mathrm{Q}x \quad \vee \quad \neg\ \mathrm{R}(x \oplus \epsilon) = \mathrm{R}x$

28: $\mathrm{R}(x \oplus \epsilon) = \mathrm{R}x$ by

25: $((\mathrm{R}x) \cdot 1) + 0 = \mathrm{R}(x \oplus \epsilon)$

26: $\neg\ ((\mathrm{R}x) \cdot 1) + 0 = \mathrm{R}(x \oplus \epsilon) \quad \vee \quad \mathrm{R}(x \oplus \epsilon) = \mathrm{R}x$

29: QEA by

27: $\neg\ \mathrm{R}(x \oplus \epsilon) = \mathrm{R}x$

28: $\mathrm{R}(x \oplus \epsilon) = \mathrm{R}x$