Proof of Theorem 195

The theorem to be proved is

$x \cdot 1 = x$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\ (x \cdot 1) = (x)]]$

Special cases of the hypothesis and previous results:

0: $\neg\ x \cdot 1 = x$ from H:x

1: $1 \cdot x = x$ from 117;x

2: $1 \cdot x = x \cdot 1$ from 105;x;1

Equality substitutions:

3: $\neg\ 1 \cdot x = x \quad \vee \quad \neg\ 1 \cdot x = x \cdot 1 \quad \vee \quad x = x \cdot 1$

Inferences:

4: $\neg\ 1 \cdot x = x \quad \vee \quad \neg\ 1 \cdot x = x \cdot 1$ by

0: $\neg\ x \cdot 1 = x$

3: $\neg\ 1 \cdot x = x \quad \vee \quad \neg\ 1 \cdot x = x \cdot 1 \quad \vee \quad x \cdot 1 = x$

5: $\neg\ 1 \cdot x = x \cdot 1$ by

1: $1 \cdot x = x$

4: $\neg\ 1 \cdot x = x \quad \vee \quad \neg\ 1 \cdot x = x \cdot 1$

6: QEA by

2: $1 \cdot x = x \cdot 1$

5: $\neg\ 1 \cdot x = x \cdot 1$