Proof of Theorem 194

The theorem to be proved is

$\epsilon \oplus x = x$ ⋆

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, (\epsilon \oplus x) = (x)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, \epsilon \oplus x = x$ from H:x

1: $\neg\, \mathrm{Q}(\epsilon \oplus x) = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{R}(\epsilon \oplus x) = \mathrm{R}x \quad \vee \quad \epsilon \oplus x = x$ from 193;$\epsilon \oplus x$;x

2: $\mathrm{Q}\epsilon = 1$ from 189

3: $\mathrm{R}\epsilon = 0$ from 189

4: $(\mathrm{Q}\epsilon) \cdot (\mathrm{Q}x) = \mathrm{Q}(\epsilon \oplus x)$ from 180;ϵ;x

5: $((\mathrm{R}\epsilon) \cdot (\mathrm{Q}x)) + (\mathrm{R}x) = \mathrm{R}(\epsilon \oplus x)$ from 180;ϵ;x

6: $1 \cdot (\mathrm{Q}x) = \mathrm{Q}x$ from 117;$\mathrm{Q}x$

7: $0 \cdot (\mathrm{Q}x) = 0$ from 103;$\mathrm{Q}x$

8: $0 + (\mathrm{R}x) = \mathrm{R}x$ from 97;$\mathrm{R}x$

Equality substitutions:

9: $\neg\, \mathrm{Q}\epsilon = 1 \quad \vee \quad \neg\, (\mathrm{Q}\epsilon) \cdot (\mathrm{Q}x) = \mathrm{Q}(\epsilon \oplus x) \quad \vee \quad (1) \cdot (\mathrm{Q}x) = \mathrm{Q}(\epsilon \oplus x)$

10: $\neg\, \mathrm{R}\epsilon = 0 \quad \vee \quad \neg\, ((\mathrm{R}\epsilon) \cdot (\mathrm{Q}x)) + (\mathrm{R}x) = \mathrm{R}(\epsilon \oplus x) \quad \vee \quad ((0) \cdot (\mathrm{Q}x)) + (\mathrm{R}x) = \mathrm{R}(\epsilon \oplus x)$

11: $\neg\, 1 \cdot (\mathrm{Q}x) = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{Q}(\epsilon \oplus x) = 1 \cdot (\mathrm{Q}x) \quad \vee \quad \mathrm{Q}(\epsilon \oplus x) = \mathrm{Q}x$

12: $\neg\, 0 \cdot (\mathrm{Q}x) = 0 \quad \vee \quad (0 \cdot (\mathrm{Q}x)) + (\mathrm{R}x) = \mathrm{R}x \quad \vee \quad \neg\, (0) + (\mathrm{R}x) = \mathrm{R}x$

13: $\neg\, (0 \cdot (\mathrm{Q}x)) + (\mathrm{R}x) = \mathrm{R}(\epsilon \oplus x) \quad \vee \quad \neg\, (0 \cdot (\mathrm{Q}x)) + (\mathrm{R}x) = \mathrm{R}x \quad \vee \quad \mathrm{R}(\epsilon \oplus x) = \mathrm{R}x$

Inferences:

14: $\neg\, \mathrm{Q}(\epsilon \oplus x) = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{R}(\epsilon \oplus x) = \mathrm{R}x$ by

0: $\neg\, \epsilon \oplus x = x$

1: $\neg\, \mathrm{Q}(\epsilon \oplus x) = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{R}(\epsilon \oplus x) = \mathrm{R}x \quad \vee \quad \epsilon \oplus x = x$

15: $\neg\, (\mathrm{Q}\epsilon) \cdot (\mathrm{Q}x) = \mathrm{Q}(\epsilon \oplus x) \quad \vee \quad \mathrm{Q}(\epsilon \oplus x) = 1 \cdot (\mathrm{Q}x)$ by

2: $\mathrm{Q}\epsilon = 1$

9: $\neg\, \mathrm{Q}\epsilon = 1 \quad \vee \quad \neg\, (\mathrm{Q}\epsilon) \cdot (\mathrm{Q}x) = \mathrm{Q}(\epsilon \oplus x) \quad \vee \quad \mathrm{Q}(\epsilon \oplus x) = 1 \cdot (\mathrm{Q}x)$

16: $\neg\, ((\mathrm{R}\epsilon) \cdot (\mathrm{Q}x)) + (\mathrm{R}x) = \mathrm{R}(\epsilon \oplus x) \quad \vee \quad (0 \cdot (\mathrm{Q}x)) + (\mathrm{R}x) = \mathrm{R}(\epsilon \oplus x)$ by

3: $\mathrm{R}\epsilon = 0$

10: $\neg\, \mathrm{R}\epsilon = 0 \quad \vee \quad \neg\, ((\mathrm{R}\epsilon) \cdot (\mathrm{Q}x)) + (\mathrm{R}x) = \mathrm{R}(\epsilon \oplus x) \quad \vee \quad (0 \cdot (\mathrm{Q}x)) + (\mathrm{R}x) = \mathrm{R}(\epsilon \oplus x)$

17: $\mathrm{Q}(\epsilon \oplus x) = 1 \cdot (\mathrm{Q}x)$ by

4: $(\mathrm{Q}\epsilon) \cdot (\mathrm{Q}x) = \mathrm{Q}(\epsilon \oplus x)$

15: $\neg\, (\mathrm{Q}\epsilon) \cdot (\mathrm{Q}x) = \mathrm{Q}(\epsilon \oplus x) \quad \vee \quad \mathrm{Q}(\epsilon \oplus x) = 1 \cdot (\mathrm{Q}x)$

18: $(0 \cdot (\mathrm{Q}x)) + (\mathrm{R}x) = \mathrm{R}(\epsilon \oplus x)$ by

5: $((\mathrm{R}\epsilon) \cdot (\mathrm{Q}x)) + (\mathrm{R}x) = \mathrm{R}(\epsilon \oplus x)$

16: $\neg\, ((\mathrm{R}\epsilon) \cdot (\mathrm{Q}x)) + (\mathrm{R}x) = \mathrm{R}(\epsilon \oplus x) \quad \vee \quad (0 \cdot (\mathrm{Q}x)) + (\mathrm{R}x) = \mathrm{R}(\epsilon \oplus x)$

19: $\neg\, \mathrm{Q}(\epsilon \oplus x) = 1 \cdot (\mathrm{Q}x) \quad \vee \quad \mathrm{Q}(\epsilon \oplus x) = \mathrm{Q}x$ by

6: $1 \cdot (\mathrm{Q}x) = \mathrm{Q}x$

11: $\neg\, 1 \cdot (\mathrm{Q}x) = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{Q}(\epsilon \oplus x) = 1 \cdot (\mathrm{Q}x) \quad \vee \quad \mathrm{Q}(\epsilon \oplus x) = \mathrm{Q}x$

20: $(0 \cdot (\mathrm{Q}x)) + (\mathrm{R}x) = \mathrm{R}x \quad \vee \quad \neg\, 0 + (\mathrm{R}x) = \mathrm{R}x$ by

7: $0 \cdot (\mathrm{Q}x) = 0$

12: $\neg\, 0 \cdot (\mathrm{Q}x) = 0 \quad \vee \quad (0 \cdot (\mathrm{Q}x)) + (\mathrm{R}x) = \mathrm{R}x \quad \vee \quad \neg\, 0 + (\mathrm{R}x) = \mathrm{R}x$

21: $(0 \cdot (\mathrm{Q}x)) + (\mathrm{R}x) = \mathrm{R}x$ by

8: $0 + (\mathrm{R}x) = \mathrm{R}x$

20: $(0 \cdot (\mathrm{Q}x)) + (\mathrm{R}x) = \mathrm{R}x \quad \vee \quad \neg\, 0 + (\mathrm{R}x) = \mathrm{R}x$

22: $\mathrm{Q}(\epsilon \oplus x) = \mathrm{Q}x$ by

17: $\mathrm{Q}(\epsilon \oplus x) = 1 \cdot (\mathrm{Q}x)$

19: $\neg\, \mathrm{Q}(\epsilon \oplus x) = 1 \cdot (\mathrm{Q}x) \quad \vee \quad \mathrm{Q}(\epsilon \oplus x) = \mathrm{Q}x$

23: $\neg\, (0 \cdot (\mathrm{Q}x)) + (\mathrm{R}x) = \mathrm{R}x \quad \vee \quad \mathrm{R}(\epsilon \oplus x) = \mathrm{R}x$ by

18: $(0 \cdot (\mathrm{Q}x)) + (\mathrm{R}x) = \mathrm{R}(\epsilon \oplus x)$

13: $\neg\, (0 \cdot (\mathrm{Q}x)) + (\mathrm{R}x) = \mathrm{R}(\epsilon \oplus x) \quad \vee \quad \neg\, (0 \cdot (\mathrm{Q}x)) + (\mathrm{R}x) = \mathrm{R}x \quad \vee \quad \mathrm{R}(\epsilon \oplus x) = \mathrm{R}x$

24: $\mathrm{R}(\epsilon \oplus x) = \mathrm{R}x$ by

21: $(0 \cdot (\mathrm{Q}x)) + (\mathrm{R}x) = \mathrm{R}x$

23: $\neg\, (0 \cdot (\mathrm{Q}x)) + (\mathrm{R}x) = \mathrm{R}x \quad \vee \quad \mathrm{R}(\epsilon \oplus x) = \mathrm{R}x$

25: $\neg\, \mathrm{R}(\epsilon \oplus x) = \mathrm{R}x$ by

22: $\mathrm{Q}(\epsilon \oplus x) = \mathrm{Q}x$

14: $\neg\, \mathrm{Q}(\epsilon \oplus x) = \mathrm{Q}x \quad \vee \quad \neg\, \mathrm{R}(\epsilon \oplus x) = \mathrm{R}x$

26: QEA by

24: $\mathrm{R}(\epsilon \oplus x) = \mathrm{R}x$

25: $\neg\, \mathrm{R}(\epsilon \oplus x) = \mathrm{R}x$