Proof of Theorem 193

The theorem to be proved is

$Qx = Qy \quad \& \quad Rx = Ry \quad \rightarrow \quad x = y$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(Qx) = (Qy)] \quad \& \quad [(Rx) = (Ry)] \quad \& \quad [\neg\,(x) = (y)]]$

Special cases of the hypothesis and previous results:

0: $Qy = Qx$ from H:x:y

1: $Ry = Rx$ from H:x:y

2: $\neg\, y = x$ from H:x:y

3: $(Qx) + (Rx) = Sx$ from 166;x

4: $(Qy) + (Ry) = Sy$ from 166;y

5: $\neg\, Sy = Sx \quad \vee \quad y = x$ from 4;x;y

Equality substitutions:

6: $\neg\, Qy = Qx \quad \vee \quad \neg\,(Qy) + (Ry) = Sy \quad \vee \quad (Qx) + (Ry) = Sy$

7: $\neg\, Ry = Rx \quad \vee \quad (Qx) + (Ry) = Sx \quad \vee \quad \neg\,(Qx) + (Rx) = Sx$

8: $\neg\,(Qx) + (Ry) = Sy \quad \vee \quad \neg\,(Qx) + (Ry) = Sx \quad \vee \quad Sy = Sx$

Inferences:

9: $\neg\,(Qy) + (Ry) = Sy \quad \vee \quad (Qx) + (Ry) = Sy$ by
0: $Qy = Qx$
6: $\neg\, Qy = Qx \quad \vee \quad \neg\,(Qy) + (Ry) = Sy \quad \vee \quad (Qx) + (Ry) = Sy$

10: $(Qx) + (Ry) = Sx \quad \vee \quad \neg\,(Qx) + (Rx) = Sx$ by
1: $Ry = Rx$
7: $\neg\, Ry = Rx \quad \vee \quad (Qx) + (Ry) = Sx \quad \vee \quad \neg\,(Qx) + (Rx) = Sx$

11: $\neg\, Sy = Sx$ by
2: $\neg\, y = x$
5: $\neg\, Sy = Sx \quad \vee \quad y = x$

12: $(\mathrm{Q}x) + (\mathrm{R}y) = \mathrm{S}x$ by

3: $(\mathrm{Q}x) + (\mathrm{R}x) = \mathrm{S}x$

10: $(\mathrm{Q}x) + (\mathrm{R}y) = \mathrm{S}x \quad \vee \quad \neg\, (\mathrm{Q}x) + (\mathrm{R}x) = \mathrm{S}x$

13: $(\mathrm{Q}x) + (\mathrm{R}y) = \mathrm{S}y$ by

4: $(\mathrm{Q}y) + (\mathrm{R}y) = \mathrm{S}y$

9: $\neg\, (\mathrm{Q}y) + (\mathrm{R}y) = \mathrm{S}y \quad \vee \quad (\mathrm{Q}x) + (\mathrm{R}y) = \mathrm{S}y$

14: $\neg\, (\mathrm{Q}x) + (\mathrm{R}y) = \mathrm{S}y \quad \vee \quad \neg\, (\mathrm{Q}x) + (\mathrm{R}y) = \mathrm{S}x$ by

11: $\neg\, \mathrm{S}y = \mathrm{S}x$

8: $\neg\, (\mathrm{Q}x) + (\mathrm{R}y) = \mathrm{S}y \quad \vee \quad \neg\, (\mathrm{Q}x) + (\mathrm{R}y) = \mathrm{S}x \quad \vee \quad \mathrm{S}y = \mathrm{S}x$

15: $\neg\, (\mathrm{Q}x) + (\mathrm{R}y) = \mathrm{S}y$ by

12: $(\mathrm{Q}x) + (\mathrm{R}y) = \mathrm{S}x$

14: $\neg\, (\mathrm{Q}x) + (\mathrm{R}y) = \mathrm{S}y \quad \vee \quad \neg\, (\mathrm{Q}x) + (\mathrm{R}y) = \mathrm{S}x$

16: QEA by

13: $(\mathrm{Q}x) + (\mathrm{R}y) = \mathrm{S}y$

15: $\neg\, (\mathrm{Q}x) + (\mathrm{R}y) = \mathrm{S}y$