Proof of Theorem 192

The theorem to be proved is

$Q\underline{1} = 2 \quad \& \quad R\underline{1} = 1$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg (Q\underline{1}) = (2) \quad \vee \quad \neg (R\underline{1}) = (1)]]$

Special cases of the hypothesis and previous results:

0: $\neg Q\underline{1} = 2 \quad \vee \quad \neg R\underline{1} = 1$ from H

1: $S(S0) = 2$ from 116

2: $S0 = 1$ from 115

3: $\underline{1} = 2$ from 187

4: $2 + 0 = 2$ from 12;2;0

5: $S(2 + 0) = 2 + (S0)$ from 12;2;0

6: 2 is a power of two from 190

7: $1 < S1$ from 125;1

8: $\neg 2+1 = S2 \quad \vee \quad \neg$ 2 is a power of two $\quad \vee \quad \neg 1 < 2 \quad \vee \quad Q2 = 2$ from 171;2;2;1

9: $\neg 2+1 = S2 \quad \vee \quad \neg$ 2 is a power of two $\quad \vee \quad \neg 1 < 2 \quad \vee \quad R2 = 1$ from 171;2;2;1

Equality substitutions:

10: $\neg S0 = 1 \quad \vee \quad \neg S(S0) = 2 \quad \vee \quad S(1) = 2$

11: $\neg S0 = 1 \quad \vee \quad \neg S(2 + 0) = 2 + (S0) \quad \vee \quad S(2 + 0) = 2 + (1)$

12: $\neg \underline{1} = 2 \quad \vee \quad Q\underline{1} = 2 \quad \vee \quad \neg Q2 = 2$

13: $\neg \underline{1} = 2 \quad \vee \quad R\underline{1} = 1 \quad \vee \quad \neg R2 = 1$

14: $\neg 2 + 0 = 2 \quad \vee \quad \neg S(2 + 0) = 2 + 1 \quad \vee \quad S(2) = 2 + 1$

15: $\neg S1 = 2 \quad \vee \quad \neg 1 < S1 \quad \vee \quad 1 < 2$

Inferences:

16: $\neg\, S0 = 1 \quad \vee \quad S1 = 2$ by

1: $S(S0) = 2$

10: $\neg\, S0 = 1 \quad \vee \quad \neg\, S(S0) = 2 \quad \vee \quad S1 = 2$

17: $\neg\, S(2+0) = 2 + (S0) \quad \vee \quad S(2+0) = 2 + 1$ by

2: $S0 = 1$

11: $\neg\, S0 = 1 \quad \vee \quad \neg\, S(2+0) = 2 + (S0) \quad \vee \quad S(2+0) = 2 + 1$

18: $S1 = 2$ by

2: $S0 = 1$

16: $\neg\, S0 = 1 \quad \vee \quad S1 = 2$

19: $Q\underline{1} = 2 \quad \vee \quad \neg\, Q2 = 2$ by

3: $\underline{1} = 2$

12: $\neg\, \underline{1} = 2 \quad \vee \quad Q\underline{1} = 2 \quad \vee \quad \neg\, Q2 = 2$

20: $R\underline{1} = 1 \quad \vee \quad \neg\, R2 = 1$ by

3: $\underline{1} = 2$

13: $\neg\, \underline{1} = 2 \quad \vee \quad R\underline{1} = 1 \quad \vee \quad \neg\, R2 = 1$

21: $\neg\, S(2+0) = 2 + 1 \quad \vee \quad 2 + 1 = S2$ by

4: $2 + 0 = 2$

14: $\neg\, 2 + 0 = 2 \quad \vee \quad \neg\, S(2+0) = 2 + 1 \quad \vee \quad 2 + 1 = S2$

22: $S(2+0) = 2 + 1$ by

5: $S(2+0) = 2 + (S0)$

17: $\neg\, S(2+0) = 2 + (S0) \quad \vee \quad S(2+0) = 2 + 1$

23: $\neg\, 2 + 1 = S2 \quad \vee \quad \neg\, 1 < 2 \quad \vee \quad Q2 = 2$ by

6: 2 is a power of two

8: $\neg\, 2 + 1 = S2 \quad \vee$ $\neg$ 2 is a power of two $\quad \vee \quad \neg\, 1 < 2 \quad \vee \quad Q2 = 2$

24: $\neg\, 2 + 1 = S2 \quad \vee \quad \neg\, 1 < 2 \quad \vee \quad R2 = 1$ by

6: 2 is a power of two

9: $\neg\, 2 + 1 = S2 \quad \vee$ $\neg$ 2 is a power of two $\quad \vee \quad \neg\, 1 < 2 \quad \vee \quad R2 = 1$

25: $\neg\, S1 = 2 \quad \vee \quad 1 < 2$ by

7: $1 < S1$

15: $\neg\, S1 = 2 \quad \vee \quad \neg\, 1 < S1 \quad \vee \quad 1 < 2$

26: $1 < 2$ by

18: $S1 = 2$

25: $\neg\, S1 = 2 \quad \vee \quad 1 < 2$

27: $2 + 1 = \mathrm{S}2$ by

22: $\mathrm{S}(2 + 0) = 2 + 1$

21: $\neg\, \mathrm{S}(2 + 0) = 2 + 1 \quad \vee \quad 2 + 1 = \mathrm{S}2$

28: $\neg\, 2 + 1 = \mathrm{S}2 \quad \vee \quad \mathrm{Q}2 = 2$ by

26: $1 < 2$

23: $\neg\, 2 + 1 = \mathrm{S}2 \quad \vee \quad \neg\, 1 < 2 \quad \vee \quad \mathrm{Q}2 = 2$

29: $\neg\, 2 + 1 = \mathrm{S}2 \quad \vee \quad \mathrm{R}2 = 1$ by

26: $1 < 2$

24: $\neg\, 2 + 1 = \mathrm{S}2 \quad \vee \quad \neg\, 1 < 2 \quad \vee \quad \mathrm{R}2 = 1$

30: $\mathrm{Q}2 = 2$ by

27: $2 + 1 = \mathrm{S}2$

28: $\neg\, 2 + 1 = \mathrm{S}2 \quad \vee \quad \mathrm{Q}2 = 2$

31: $\mathrm{R}2 = 1$ by

27: $2 + 1 = \mathrm{S}2$

29: $\neg\, 2 + 1 = \mathrm{S}2 \quad \vee \quad \mathrm{R}2 = 1$

32: $\mathrm{Q}\underline{1} = 2$ by

30: $\mathrm{Q}2 = 2$

19: $\mathrm{Q}\underline{1} = 2 \quad \vee \quad \neg\, \mathrm{Q}2 = 2$

33: $\mathrm{R}\underline{1} = 1$ by

31: $\mathrm{R}2 = 1$

20: $\mathrm{R}\underline{1} = 1 \quad \vee \quad \neg\, \mathrm{R}2 = 1$

34: $\neg\, \mathrm{R}\underline{1} = 1$ by

32: $\mathrm{Q}\underline{1} = 2$

0: $\neg\, \mathrm{Q}\underline{1} = 2 \quad \vee \quad \neg\, \mathrm{R}\underline{1} = 1$

35: QEA by

33: $\mathrm{R}\underline{1} = 1$

34: $\neg\, \mathrm{R}\underline{1} = 1$