Proof of Theorem 191

The theorem to be proved is

$Q\underline{0} = 2 \quad \& \quad R\underline{0} = 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg (Q\underline{0}) = (2) \quad \vee \quad \neg (R\underline{0}) = (0)]]$

Special cases of the hypothesis and previous results:

0: $\neg Q\underline{0} = 2 \quad \vee \quad \neg R\underline{0} = 0$ from H

1: $S(S0) = 2$ from 116

2: $S0 = 1$ from 115

3: $\underline{0} = 1$ from 186

4: $2 + 0 = 2$ from 12;2

5: 2 is a power of two from 190

6: $0 < S0$ from 125;0

7: $1 < S1$ from 125;1

8: $\neg 0 < 1 \quad \vee \quad 0 \leq 1$ from $56^{\rightarrow}$;0;1

9: $\neg 0 \leq 1 \quad \vee \quad \neg 1 < 2 \quad \vee \quad 0 < 2$ from 122;0;1;2

10: $\neg 2{+}0 = S1$ $\vee$ $\neg$ 2 is a power of two $\vee$ $\neg 0 < 2$ $\vee$ $Q1 = 2$ from 171;1;2;0

11: $\neg 2{+}0 = S1$ $\vee$ $\neg$ 2 is a power of two $\vee$ $\neg 0 < 2$ $\vee$ $R1 = 0$ from 171;1;2;0

Equality substitutions:

12: $\neg S0 = 1 \quad \vee \quad \neg S(S0) = 2 \quad \vee \quad S(1) = 2$

13: $\neg S0 = 1 \quad \vee \quad \neg 0 < S0 \quad \vee \quad 0 < 1$

14: $\neg \underline{0} = 1 \quad \vee \quad Q\underline{0} = 2 \quad \vee \quad \neg Q1 = 2$

15: $\neg \underline{0} = 1 \quad \vee \quad R\underline{0} = 0 \quad \vee \quad \neg R1 = 0$

16: $\neg 2 + 0 = 2 \quad \vee \quad 2 + 0 = S1 \quad \vee \quad \neg 2 = S1$

17: $\neg S1 = 2 \quad \vee \quad \neg 1 < S1 \quad \vee \quad 1 < 2$

Inferences:

18: $\neg\, \mathrm{S}0 = 1 \quad \vee \quad \mathrm{S}1 = 2$ by

1: $\mathrm{S}(\mathrm{S}0) = 2$

12: $\neg\, \mathrm{S}0 = 1 \quad \vee \quad \neg\, \mathrm{S}(\mathrm{S}0) = 2 \quad \vee \quad \mathrm{S}1 = 2$

19: $\neg\, 0 < \mathrm{S}0 \quad \vee \quad 0 < 1$ by

2: $\mathrm{S}0 = 1$

13: $\neg\, \mathrm{S}0 = 1 \quad \vee \quad \neg\, 0 < \mathrm{S}0 \quad \vee \quad 0 < 1$

20: $\mathrm{S}1 = 2$ by

2: $\mathrm{S}0 = 1$

18: $\neg\, \mathrm{S}0 = 1 \quad \vee \quad \mathrm{S}1 = 2$

21: $\mathrm{Q}\underline{0} = 2 \quad \vee \quad \neg\, \mathrm{Q}1 = 2$ by

3: $\underline{0} = 1$

14: $\neg\, \underline{0} = 1 \quad \vee \quad \mathrm{Q}\underline{0} = 2 \quad \vee \quad \neg\, \mathrm{Q}1 = 2$

22: $\mathrm{R}\underline{0} = 0 \quad \vee \quad \neg\, \mathrm{R}1 = 0$ by

3: $\underline{0} = 1$

15: $\neg\, \underline{0} = 1 \quad \vee \quad \mathrm{R}\underline{0} = 0 \quad \vee \quad \neg\, \mathrm{R}1 = 0$

23: $2 + 0 = \mathrm{S}1 \quad \vee \quad \neg\, \mathrm{S}1 = 2$ by

4: $2 + 0 = 2$

16: $\neg\, 2 + 0 = 2 \quad \vee \quad 2 + 0 = \mathrm{S}1 \quad \vee \quad \neg\, \mathrm{S}1 = 2$

24: $\neg\, 2 + 0 = \mathrm{S}1 \quad \vee \quad \neg\, 0 < 2 \quad \vee \quad \mathrm{Q}1 = 2$ by

5: 2 is a power of two

10: $\neg\, 2 + 0 = \mathrm{S}1 \quad \vee$ $\neg$ 2 is a power of two $\quad \vee \quad \neg\, 0 < 2 \quad \vee \quad \mathrm{Q}1 = 2$

25: $\neg\, 2 + 0 = \mathrm{S}1 \quad \vee \quad \neg\, 0 < 2 \quad \vee \quad \mathrm{R}1 = 0$ by

5: 2 is a power of two

11: $\neg\, 2 + 0 = \mathrm{S}1 \quad \vee$ $\neg$ 2 is a power of two $\quad \vee \quad \neg\, 0 < 2 \quad \vee \quad \mathrm{R}1 = 0$

26: $0 < 1$ by

6: $0 < \mathrm{S}0$

19: $\neg\, 0 < \mathrm{S}0 \quad \vee \quad 0 < 1$

27: $\neg\, \mathrm{S}1 = 2 \quad \vee \quad 1 < 2$ by

7: $1 < \mathrm{S}1$

17: $\neg\, \mathrm{S}1 = 2 \quad \vee \quad \neg\, 1 < \mathrm{S}1 \quad \vee \quad 1 < 2$

28: $2 + 0 = \mathrm{S}1$ by

20: $\mathrm{S}1 = 2$

23: $2 + 0 = \mathrm{S}1 \quad \vee \quad \neg\, \mathrm{S}1 = 2$

29: $1 < 2$ by

20: $S1 = 2$

27: $\neg\, S1 = 2 \;\vee\; 1 < 2$

30: $0 \leq 1$ by

26: $0 < 1$

8: $\neg\, 0 < 1 \;\vee\; 0 \leq 1$

31: $\neg\, 0 < 2 \;\vee\; Q1 = 2$ by

28: $2 + 0 = S1$

24: $\neg\, 2 + 0 = S1 \;\vee\; \neg\, 0 < 2 \;\vee\; Q1 = 2$

32: $\neg\, 0 < 2 \;\vee\; R1 = 0$ by

28: $2 + 0 = S1$

25: $\neg\, 2 + 0 = S1 \;\vee\; \neg\, 0 < 2 \;\vee\; R1 = 0$

33: $\neg\, 0 \leq 1 \;\vee\; 0 < 2$ by

29: $1 < 2$

9: $\neg\, 0 \leq 1 \;\vee\; \neg\, 1 < 2 \;\vee\; 0 < 2$

34: $0 < 2$ by

30: $0 \leq 1$

33: $\neg\, 0 \leq 1 \;\vee\; 0 < 2$

35: $Q1 = 2$ by

34: $0 < 2$

31: $\neg\, 0 < 2 \;\vee\; Q1 = 2$

36: $R1 = 0$ by

34: $0 < 2$

32: $\neg\, 0 < 2 \;\vee\; R1 = 0$

37: $Q\underline{0} = 2$ by

35: $Q1 = 2$

21: $Q\underline{0} = 2 \;\vee\; \neg\, Q1 = 2$

38: $R\underline{0} = 0$ by

36: $R1 = 0$

22: $R\underline{0} = 0 \;\vee\; \neg\, R1 = 0$

39: $\neg\, R\underline{0} = 0$ by

37: $Q\underline{0} = 2$

0: $\neg\, Q\underline{0} = 2 \;\vee\; \neg\, R\underline{0} = 0$

40: QEA by

38: $\mathrm{R}\underline{0} = 0$

39: $\neg\ \mathrm{R}\underline{0} = 0$