Proof of Theorem 190

The theorem to be proved is

2 is a power of two

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) [[¬ (2) is a power of two]]

Special cases of the hypothesis and previous results:

0: ¬ 2 is a power of two from H

1: $S0 = 1$ from 115

2: $2 \uparrow 1$ is a power of two from 131;1

3: $2 \uparrow 0 = 1$ from 126;2;0

4: $2 \cdot (2 \uparrow 0) = 2 \uparrow (S0)$ from 126;2;0

5: $1 \cdot 2 = 2$ from 117;2

6: $2 \cdot 1 = 1 \cdot 2$ from 105;2;1

Equality substitutions:

7: $\neg\, S0 = 1 \quad \vee \quad \neg\, 2 \uparrow (S0) = 2 \quad \vee \quad 2 \uparrow (1) = 2$

8: $\neg\, 2 \uparrow 0 = 1 \quad \vee \quad \neg\, 2 \cdot (2 \uparrow 0) = 2 \uparrow (S0) \quad \vee \quad 2 \cdot (1) = 2 \uparrow (S0)$

9: $\neg\, 1 \cdot 2 = 2 \quad \vee \quad \neg\, 2 \cdot 1 = 1 \cdot 2 \quad \vee \quad 2 \cdot 1 = 2$

10: $\neg\, 2 \uparrow (S0) = 2 \cdot 1 \quad \vee \quad 2 \uparrow (S0) = 2 \quad \vee \quad \neg\, 2 \cdot 1 = 2$

11: $\neg\, 2 \uparrow 1 = 2 \quad \vee \quad \neg\, 2 \uparrow 1$ is a power of two $\quad \vee \quad 2$ is a power of two

Inferences:

12: $\neg\, 2 \uparrow 1 = 2 \quad \vee \quad \neg\, 2 \uparrow 1$ is a power of two by

0: ¬ 2 is a power of two

11: $\neg\, 2 \uparrow 1 = 2 \quad \vee \quad \neg\, 2 \uparrow 1$ is a power of two $\quad \vee \quad 2$ is a power of two

13: $\neg\, 2 \uparrow (S0) = 2 \quad \vee \quad 2 \uparrow 1 = 2$ by

1: $S0 = 1$

7: $\neg\, S0 = 1 \quad \vee \quad \neg\, 2 \uparrow (S0) = 2 \quad \vee \quad 2 \uparrow 1 = 2$

14: $\neg\, 2 \uparrow 1 = 2$ by

2: $2 \uparrow 1$ is a power of two

12: $\neg\, 2 \uparrow 1 = 2 \quad \vee \quad \neg\, 2 \uparrow 1$ is a power of two

15: $\neg\, 2 \cdot (2 \uparrow 0) = 2 \uparrow (\mathrm{S}0) \quad \vee \quad 2 \uparrow (\mathrm{S}0) = 2 \cdot 1$ by

3: $2 \uparrow 0 = 1$

8: $\neg\, 2 \uparrow 0 = 1 \quad \vee \quad \neg\, 2 \cdot (2 \uparrow 0) = 2 \uparrow (\mathrm{S}0) \quad \vee \quad 2 \uparrow (\mathrm{S}0) = 2 \cdot 1$

16: $2 \uparrow (\mathrm{S}0) = 2 \cdot 1$ by

4: $2 \cdot (2 \uparrow 0) = 2 \uparrow (\mathrm{S}0)$

15: $\neg\, 2 \cdot (2 \uparrow 0) = 2 \uparrow (\mathrm{S}0) \quad \vee \quad 2 \uparrow (\mathrm{S}0) = 2 \cdot 1$

17: $\neg\, 2 \cdot 1 = 1 \cdot 2 \quad \vee \quad 2 \cdot 1 = 2$ by

5: $1 \cdot 2 = 2$

9: $\neg\, 1 \cdot 2 = 2 \quad \vee \quad \neg\, 2 \cdot 1 = 1 \cdot 2 \quad \vee \quad 2 \cdot 1 = 2$

18: $2 \cdot 1 = 2$ by

6: $2 \cdot 1 = 1 \cdot 2$

17: $\neg\, 2 \cdot 1 = 1 \cdot 2 \quad \vee \quad 2 \cdot 1 = 2$

19: $\neg\, 2 \uparrow (\mathrm{S}0) = 2$ by

14: $\neg\, 2 \uparrow 1 = 2$

13: $\neg\, 2 \uparrow (\mathrm{S}0) = 2 \quad \vee \quad 2 \uparrow 1 = 2$

20: $2 \uparrow (\mathrm{S}0) = 2 \quad \vee \quad \neg\, 2 \cdot 1 = 2$ by

16: $2 \uparrow (\mathrm{S}0) = 2 \cdot 1$

10: $\neg\, 2 \uparrow (\mathrm{S}0) = 2 \cdot 1 \quad \vee \quad 2 \uparrow (\mathrm{S}0) = 2 \quad \vee \quad \neg\, 2 \cdot 1 = 2$

21: $2 \uparrow (\mathrm{S}0) = 2$ by

18: $2 \cdot 1 = 2$

20: $2 \uparrow (\mathrm{S}0) = 2 \quad \vee \quad \neg\, 2 \cdot 1 = 2$

22: QEA by

19: $\neg\, 2 \uparrow (\mathrm{S}0) = 2$

21: $2 \uparrow (\mathrm{S}0) = 2$