Proof of Theorem 18b

The theorem to be proved is

$\mathrm{S}x - \mathrm{S}0 = x - 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, ((\mathrm{S}x) - (\mathrm{S}0)) = (x - 0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, (\mathrm{S}x) - (\mathrm{S}0) = x - 0$ from H:x

1: $(\mathrm{S}x) - 0 = \mathrm{S}x$ from 17;Sx;0

2: $\mathrm{P}((\mathrm{S}x) - 0) = (\mathrm{S}x) - (\mathrm{S}0)$ from 17;Sx;0

3: $\mathrm{P}(\mathrm{S}x) = x$ from 16;x

4: $x - 0 = x$ from 17;x

Equality substitutions:

5: $\neg\, (\mathrm{S}x) - 0 = \mathrm{S}x \quad \vee \quad \mathrm{P}((\mathrm{S}x) - 0) = x \quad \vee \quad \neg\, \mathrm{P}(\mathrm{S}x) = x$

6: $\neg\, \mathrm{P}((\mathrm{S}x) - 0) = (\mathrm{S}x) - (\mathrm{S}0) \quad \vee \quad \neg\, \mathrm{P}((\mathrm{S}x) - 0) = x - 0 \quad \vee \quad (\mathrm{S}x) - (\mathrm{S}0) = x - 0$

7: $\neg\, x - 0 = x \quad \vee \quad \mathrm{P}((\mathrm{S}x) - 0) = x - 0 \quad \vee \quad \neg\, \mathrm{P}((\mathrm{S}x) - 0) = x$

Inferences:

8: $\neg\, \mathrm{P}((\mathrm{S}x) - 0) = (\mathrm{S}x) - (\mathrm{S}0) \quad \vee \quad \neg\, \mathrm{P}((\mathrm{S}x) - 0) = x - 0$ by
0: $\neg\, (\mathrm{S}x) - (\mathrm{S}0) = x - 0$
6: $\neg\, \mathrm{P}((\mathrm{S}x) - 0) = (\mathrm{S}x) - (\mathrm{S}0) \quad \vee \quad \neg\, \mathrm{P}((\mathrm{S}x) - 0) = x - 0 \quad \vee \quad (\mathrm{S}x) - (\mathrm{S}0) = x - 0$

9: $\mathrm{P}((\mathrm{S}x) - 0) = x \quad \vee \quad \neg\, \mathrm{P}(\mathrm{S}x) = x$ by
1: $(\mathrm{S}x) - 0 = \mathrm{S}x$
5: $\neg\, (\mathrm{S}x) - 0 = \mathrm{S}x \quad \vee \quad \mathrm{P}((\mathrm{S}x) - 0) = x \quad \vee \quad \neg\, \mathrm{P}(\mathrm{S}x) = x$

10: $\neg\, \mathrm{P}((\mathrm{S}x) - 0) = x - 0$ by
2: $\mathrm{P}((\mathrm{S}x) - 0) = (\mathrm{S}x) - (\mathrm{S}0)$
8: $\neg\, \mathrm{P}((\mathrm{S}x) - 0) = (\mathrm{S}x) - (\mathrm{S}0) \quad \vee \quad \neg\, \mathrm{P}((\mathrm{S}x) - 0) = x - 0$

11: $\mathrm{P}((\mathrm{S}x) - 0) = x$ by

3: $\mathrm{P}(\mathrm{S}x) = x$

9: $\mathrm{P}((\mathrm{S}x) - 0) = x \quad \vee \quad \neg\, \mathrm{P}(\mathrm{S}x) = x$

12: $\mathrm{P}((\mathrm{S}x) - 0) = x - 0 \quad \vee \quad \neg\, \mathrm{P}((\mathrm{S}x) - 0) = x$ by

4: $x - 0 = x$

7: $\neg\, x - 0 = x \quad \vee \quad \mathrm{P}((\mathrm{S}x) - 0) = x - 0 \quad \vee \quad \neg\, \mathrm{P}((\mathrm{S}x) - 0) = x$

13: $\neg\, \mathrm{P}((\mathrm{S}x) - 0) = x$ by

10: $\neg\, \mathrm{P}((\mathrm{S}x) - 0) = x - 0$

12: $\mathrm{P}((\mathrm{S}x) - 0) = x - 0 \quad \vee \quad \neg\, \mathrm{P}((\mathrm{S}x) - 0) = x$

14: QEA by

11: $\mathrm{P}((\mathrm{S}x) - 0) = x$

13: $\neg\, \mathrm{P}((\mathrm{S}x) - 0) = x$