Proof of Theorem 189

The theorem to be proved is

$Q\epsilon = 1 \quad \& \quad R\epsilon = 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(Q\epsilon) = (1) \quad \vee \quad \neg\,(R\epsilon) = (0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, Q\epsilon = 1 \quad \vee \quad \neg\, R\epsilon = 0$ from H

1: $S0 = 1$ from 115

2: $\epsilon = 0$ from 185

3: $1 + 0 = 1$ from 12;1

4: $0 < S0$ from 125;0

5: 1 is a power of two from 130

6: $\neg\, 1{+}0 = S0 \quad \vee \quad \neg$ 1 is a power of two $\quad \vee \quad \neg\, 0 < 1 \quad \vee \quad Q0 = 1$ from 171;0;1;0

7: $\neg\, 1{+}0 = S0 \quad \vee \quad \neg$ 1 is a power of two $\quad \vee \quad \neg\, 0 < 1 \quad \vee \quad R0 = 0$ from 171;0;1;0

Equality substitutions:

8: $\neg\, S0 = 1 \quad \vee \quad \neg\, 0 < S0 \quad \vee \quad 0 < 1$

9: $\neg\, S0 = 1 \quad \vee \quad 1 + 0 = S0 \quad \vee \quad \neg\, 1 + 0 = 1$

10: $\neg\, \epsilon = 0 \quad \vee \quad Q\epsilon = 1 \quad \vee \quad \neg\, Q0 = 1$

11: $\neg\, \epsilon = 0 \quad \vee \quad R\epsilon = 0 \quad \vee \quad \neg\, R0 = 0$

Inferences:

12: $\neg\, 0 < S0 \quad \vee \quad 0 < 1$ by
1: $S0 = 1$
8: $\neg\, S0 = 1 \quad \vee \quad \neg\, 0 < S0 \quad \vee \quad 0 < 1$

13: $1 + 0 = S0 \quad \vee \quad \neg\, 1 + 0 = 1$ by
1: $S0 = 1$
9: $\neg\, S0 = 1 \quad \vee \quad 1 + 0 = S0 \quad \vee \quad \neg\, 1 + 0 = 1$

14: $Q\epsilon = 1 \quad \vee \quad \neg\, Q0 = 1 \qquad$ by

2: $\epsilon = 0$

10: $\neg\, \epsilon = 0 \quad \vee \quad Q\epsilon = 1 \quad \vee \quad \neg\, Q0 = 1$

15: $R\epsilon = 0 \quad \vee \quad \neg\, R0 = 0 \qquad$ by

2: $\epsilon = 0$

11: $\neg\, \epsilon = 0 \quad \vee \quad R\epsilon = 0 \quad \vee \quad \neg\, R0 = 0$

16: $1 + 0 = S0 \qquad$ by

3: $1 + 0 = 1$

13: $1 + 0 = S0 \quad \vee \quad \neg\, 1 + 0 = 1$

17: $0 < 1 \qquad$ by

4: $0 < S0$

12: $\neg\, 0 < S0 \quad \vee \quad 0 < 1$

18: $\neg\, 1 + 0 = S0 \quad \vee \quad \neg\, 0 < 1 \quad \vee \quad Q0 = 1 \qquad$ by

5: 1 is a power of two

6: $\neg\, 1 + 0 = S0 \quad \vee \quad \neg$ 1 is a power of two $\quad \vee \quad \neg\, 0 < 1 \quad \vee \quad Q0 = 1$

19: $\neg\, 1 + 0 = S0 \quad \vee \quad \neg\, 0 < 1 \quad \vee \quad R0 = 0 \qquad$ by

5: 1 is a power of two

7: $\neg\, 1 + 0 = S0 \quad \vee \quad \neg$ 1 is a power of two $\quad \vee \quad \neg\, 0 < 1 \quad \vee \quad R0 = 0$

20: $\neg\, 0 < 1 \quad \vee \quad Q0 = 1 \qquad$ by

16: $1 + 0 = S0$

18: $\neg\, 1 + 0 = S0 \quad \vee \quad \neg\, 0 < 1 \quad \vee \quad Q0 = 1$

21: $\neg\, 0 < 1 \quad \vee \quad R0 = 0 \qquad$ by

16: $1 + 0 = S0$

19: $\neg\, 1 + 0 = S0 \quad \vee \quad \neg\, 0 < 1 \quad \vee \quad R0 = 0$

22: $Q0 = 1 \qquad$ by

17: $0 < 1$

20: $\neg\, 0 < 1 \quad \vee \quad Q0 = 1$

23: $R0 = 0 \qquad$ by

17: $0 < 1$

21: $\neg\, 0 < 1 \quad \vee \quad R0 = 0$

24: $Q\epsilon = 1 \qquad$ by

22: $Q0 = 1$

14: $Q\epsilon = 1 \quad \vee \quad \neg\, Q0 = 1$

25: $\mathrm{R}\epsilon = 0$ by

23: $\mathrm{R}0 = 0$

15: $\mathrm{R}\epsilon = 0 \quad \vee \quad \neg\, \mathrm{R}0 = 0$

26: $\neg\, \mathrm{R}\epsilon = 0$ by

24: $\mathrm{Q}\epsilon = 1$

0: $\neg\, \mathrm{Q}\epsilon = 1 \quad \vee \quad \neg\, \mathrm{R}\epsilon = 0$

27: QEA by

25: $\mathrm{R}\epsilon = 0$

26: $\neg\, \mathrm{R}\epsilon = 0$