Proof of Theorem 183

The theorem to be proved is

$x \oplus (y \oplus z) = (x \oplus y) \oplus z$ ⋆

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\ (x \oplus (y \oplus z)) = ((x \oplus y) \oplus z)]]$

Special cases of the hypothesis and previous results:

0: $\neg\ x \oplus (y \oplus z) = (x \oplus y) \oplus z$ from H:x:y:z

1: $Q(x \oplus (y \oplus z)) = Q((x \oplus y) \oplus z)$ from 181;x;y;z

2: $R(x \oplus (y \oplus z)) = R((x \oplus y) \oplus z)$ from 182;x;y;z

3: $(Q(x \oplus (y \oplus z))) + (R(x \oplus (y \oplus z))) = S(x \oplus (y \oplus z))$ from 166;$x \oplus (y \oplus z)$

4: $(Q((x \oplus y) \oplus z)) + (R((x \oplus y) \oplus z)) = S((x \oplus y) \oplus z)$ from 166;$(x \oplus y) \oplus z$

5: $\neg\ S(x \oplus (y \oplus z)) = S((x \oplus y) \oplus z) \ \vee\ x \oplus (y \oplus z) = (x \oplus y) \oplus z$ from 4;$(x \oplus y) \oplus z$;$x \oplus (y \oplus z)$

Equality substitutions:

6: $\neg\ Q(x \oplus (y \oplus z)) = Q((x \oplus y) \oplus z) \ \vee\ (Q(x \oplus (y \oplus z))) + (R((x \oplus y) \oplus z)) = S((x \oplus y) \oplus z) \ \vee\ \neg\ (Q((x \oplus y) \oplus z)) + (R((x \oplus y) \oplus z)) = S((x \oplus y) \oplus z)$

7: $\neg\ R(x \oplus (y \oplus z)) = R((x \oplus y) \oplus z) \ \vee\ \neg\ (Q(x \oplus (y \oplus z))) + (R(x \oplus (y \oplus z))) = S(x \oplus (y \oplus z)) \ \vee\ (Q(x \oplus (y \oplus z))) + (R((x \oplus y) \oplus z)) = S(x \oplus (y \oplus z))$

8: $\neg\ (Q(x \oplus (y \oplus z))) + (R((x \oplus y) \oplus z)) = S(x \oplus (y \oplus z)) \ \vee\ \neg\ (Q(x \oplus (y \oplus z))) + (R((x \oplus y) \oplus z)) = S((x \oplus y) \oplus z) \ \vee\ S(x \oplus (y \oplus z)) = S((x \oplus y) \oplus z)$

Inferences:

9: $\neg\ S(x \oplus (y \oplus z)) = S((x \oplus y) \oplus z)$ by

0: $\neg\ x \oplus (y \oplus z) = (x \oplus y) \oplus z$

5: $\neg\ S(x \oplus (y \oplus z)) = S((x \oplus y) \oplus z) \ \vee\ x \oplus (y \oplus z) = (x \oplus y) \oplus z$

10: $(Q(x \oplus (y \oplus z))) + (R((x \oplus y) \oplus z)) = S((x \oplus y) \oplus z) \ \vee\ \neg\ (Q((x \oplus y) \oplus z)) + (R((x \oplus y) \oplus z)) = S((x \oplus y) \oplus z)$ by

1: $Q(x \oplus (y \oplus z)) = Q((x \oplus y) \oplus z)$

6: $\neg\, \mathrm{Q}(x{\oplus}(y{\oplus}z)) = \mathrm{Q}((x{\oplus}y){\oplus}z) \quad\vee\quad (\mathrm{Q}(x{\oplus}(y{\oplus}z))){+}(\mathrm{R}((x{\oplus}y){\oplus}z)) = \mathrm{S}((x{\oplus}y){\oplus}z) \quad\vee\quad \neg\, (\mathrm{Q}((x \oplus y) \oplus z)) + (\mathrm{R}((x \oplus y) \oplus z)) = \mathrm{S}((x \oplus y) \oplus z)$

11: $\neg\, (\mathrm{Q}(x \oplus (y \oplus z))) + (\mathrm{R}(x \oplus (y \oplus z))) = \mathrm{S}(x \oplus (y \oplus z)) \quad\vee\quad (\mathrm{Q}(x \oplus (y \oplus z))) + (\mathrm{R}((x \oplus y) \oplus z)) = \mathrm{S}(x \oplus (y \oplus z))$ by

2: $\mathrm{R}(x \oplus (y \oplus z)) = \mathrm{R}((x \oplus y) \oplus z)$

7: $\neg\, \mathrm{R}(x \oplus (y \oplus z)) = \mathrm{R}((x \oplus y) \oplus z) \quad\vee\quad \neg\, (\mathrm{Q}(x \oplus (y \oplus z))) + (\mathrm{R}(x \oplus (y \oplus z))) = \mathrm{S}(x \oplus (y \oplus z)) \quad\vee\quad (\mathrm{Q}(x \oplus (y \oplus z))) + (\mathrm{R}((x \oplus y) \oplus z)) = \mathrm{S}(x \oplus (y \oplus z))$

12: $(\mathrm{Q}(x \oplus (y \oplus z))) + (\mathrm{R}((x \oplus y) \oplus z)) = \mathrm{S}(x \oplus (y \oplus z))$ by

3: $(\mathrm{Q}(x \oplus (y \oplus z))) + (\mathrm{R}(x \oplus (y \oplus z))) = \mathrm{S}(x \oplus (y \oplus z))$

11: $\neg\, (\mathrm{Q}(x \oplus (y \oplus z))) + (\mathrm{R}(x \oplus (y \oplus z))) = \mathrm{S}(x \oplus (y \oplus z)) \quad\vee\quad (\mathrm{Q}(x \oplus (y \oplus z))) + (\mathrm{R}((x \oplus y) \oplus z)) = \mathrm{S}(x \oplus (y \oplus z))$

13: $(\mathrm{Q}(x \oplus (y \oplus z))) + (\mathrm{R}((x \oplus y) \oplus z)) = \mathrm{S}((x \oplus y) \oplus z)$ by

4: $(\mathrm{Q}((x \oplus y) \oplus z)) + (\mathrm{R}((x \oplus y) \oplus z)) = \mathrm{S}((x \oplus y) \oplus z)$

10: $(\mathrm{Q}(x \oplus (y \oplus z))) + (\mathrm{R}((x \oplus y) \oplus z)) = \mathrm{S}((x \oplus y) \oplus z) \quad\vee\quad \neg\, (\mathrm{Q}((x \oplus y) \oplus z)) + (\mathrm{R}((x \oplus y) \oplus z)) = \mathrm{S}((x \oplus y) \oplus z)$

14: $\neg\, (\mathrm{Q}(x \oplus (y \oplus z))) + (\mathrm{R}((x \oplus y) \oplus z)) = \mathrm{S}(x \oplus (y \oplus z)) \quad\vee\quad \neg\, (\mathrm{Q}(x \oplus (y \oplus z))) + (\mathrm{R}((x \oplus y) \oplus z)) = \mathrm{S}((x \oplus y) \oplus z)$ by

9: $\neg\, \mathrm{S}(x \oplus (y \oplus z)) = \mathrm{S}((x \oplus y) \oplus z)$

8: $\neg\, (\mathrm{Q}(x \oplus (y \oplus z))) + (\mathrm{R}((x \oplus y) \oplus z)) = \mathrm{S}(x \oplus (y \oplus z)) \quad\vee\quad \neg\, (\mathrm{Q}(x \oplus (y \oplus z))) + (\mathrm{R}((x \oplus y) \oplus z)) = \mathrm{S}((x \oplus y) \oplus z) \quad\vee\quad \mathrm{S}(x \oplus (y \oplus z)) = \mathrm{S}((x \oplus y) \oplus z)$

15: $\neg\, (\mathrm{Q}(x \oplus (y \oplus z))) + (\mathrm{R}((x \oplus y) \oplus z)) = \mathrm{S}((x \oplus y) \oplus z)$ by

12: $(\mathrm{Q}(x \oplus (y \oplus z))) + (\mathrm{R}((x \oplus y) \oplus z)) = \mathrm{S}(x \oplus (y \oplus z))$

14: $\neg\, (\mathrm{Q}(x \oplus (y \oplus z))) + (\mathrm{R}((x \oplus y) \oplus z)) = \mathrm{S}(x \oplus (y \oplus z)) \quad\vee\quad \neg\, (\mathrm{Q}(x \oplus (y \oplus z))) + (\mathrm{R}((x \oplus y) \oplus z)) = \mathrm{S}((x \oplus y) \oplus z)$

16: QEA by

13: $(\mathrm{Q}(x \oplus (y \oplus z))) + (\mathrm{R}((x \oplus y) \oplus z)) = \mathrm{S}((x \oplus y) \oplus z)$

15: $\neg\, (\mathrm{Q}(x \oplus (y \oplus z))) + (\mathrm{R}((x \oplus y) \oplus z)) = \mathrm{S}((x \oplus y) \oplus z)$