Proof of Theorem 180

The theorem to be proved is

$Q(x \oplus y) = Qx \cdot Qy \quad \& \quad R(x \oplus y) = Rx \cdot Qy + Ry$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, (Q(x \oplus y)) = ((Qx) \cdot (Qy)) \quad \vee \quad \neg\, (R(x \oplus y)) = (((Rx) \cdot (Qy)) + (Ry))]]$

Special cases of the hypothesis and previous results:

0: $\neg\, (Qx) \cdot (Qy) = Q(x \oplus y) \quad \vee \quad \neg\, ((Rx) \cdot (Qy)) + (Ry) = R(x \oplus y)$ from H:x:y

1: $((Sx) \cdot (Qy)) + (Ry) = S(x \oplus y)$ from 179;x;y

2: $(Qx) + (Rx) = Sx$ from 166;x

3: Qx is a power of two from 166;x

4: Qy is a power of two from 166;y

5: $((Qx) \cdot (Qy)) + ((Rx) \cdot (Qy)) = ((Qx) + (Rx)) \cdot (Qy)$ from 106;Qx;Rx;Qy

6: $((Qx) \cdot (Qy)) + (((Rx) \cdot (Qy)) + (Ry)) = (((Qx) \cdot (Qy)) + ((Rx) \cdot (Qy))) + (Ry)$ from 72;$Qx) \cdot (Qy$;$((Rx) \cdot (Qy))$;Ry

7: $\neg\, Qx$ is a power of two $\vee$ $\neg\, Qy$ is a power of two $\vee$ $(Qx) \cdot (Qy)$ is a power of two from 177;Qx;Qy

8: $((Rx) \cdot (Qy)) + (Ry) < (Qx) \cdot (Qy)$ from 176;x;y

9: $\neg\, ((Qx) \cdot (Qy)) + (((Rx) \cdot (Qy)) + (Ry)) = S(x \oplus y)$ $\vee$ $\neg\, (Qx) \cdot (Qy)$ is a power of two $\vee$ $\neg\, ((Rx) \cdot (Qy)) + (Ry) < (Qx) \cdot (Qy)$ $\vee$ $(Qx) \cdot (Qy) = Q(x \oplus y)$ from 171;$x \oplus y$;$(Qx) \cdot (Qy)$;$(Rx) \cdot (Qy)) + (Ry$

10: $\neg\, ((Qx) \cdot (Qy)) + (((Rx) \cdot (Qy)) + (Ry)) = S(x \oplus y)$ $\vee$ $\neg\, (Qx) \cdot (Qy)$ is a power of two $\vee$ $\neg\, ((Rx) \cdot (Qy)) + (Ry) < (Qx) \cdot (Qy)$ $\vee$ $((Rx) \cdot (Qy)) + (Ry) = R(x \oplus y)$ from 171;$x \oplus y$;$(Qx) \cdot (Qy)$;$(Rx) \cdot (Qy)) + (Ry$

Equality substitutions:

11: $\neg\, (Qx) + (Rx) = Sx \quad \vee \quad (((Qx) + (Rx)) \cdot (Qy)) + (Ry) = S(x \oplus y) \quad \vee \quad \neg\, ((Sx) \cdot (Qy)) + (Ry) = S(x \oplus y)$

12: $\neg\, ((Qx) \cdot (Qy)) + ((Rx) \cdot (Qy)) = ((Qx) + (Rx)) \cdot (Qy) \quad \vee \quad \neg\, ((Qx) \cdot (Qy)) + (((Rx) \cdot (Qy)) + (Ry)) = (((Qx) \cdot (Qy)) + ((Rx) \cdot (Qy))) + (Ry) \quad \vee \quad ((Qx) \cdot (Qy)) + (((Rx) \cdot (Qy)) + (Ry)) = (((Qx) + (Rx)) \cdot (Qy)) + (Ry)$

13: $\neg\, ((Qx) \cdot (Qy)) + (((Rx) \cdot (Qy)) + (Ry)) = (((Qx) + (Rx)) \cdot (Qy)) + (Ry) \quad \vee \quad ((Qx) \cdot (Qy)) + (((Rx) \cdot (Qy)) + (Ry)) = S(x \oplus y) \quad \vee \quad \neg\, (((Qx) + (Rx)) \cdot (Qy)) + (Ry) = S(x \oplus y)$

Inferences:

14: $\neg$ (Qx) + (Rx) = Sx $\vee$ (((Qx) + (Rx)) $\cdot$ (Qy)) + (Ry) = S($x \oplus y$) by

1: ((Sx) $\cdot$ (Qy)) + (Ry) = S($x \oplus y$)

11: $\neg$ (Qx) + (Rx) = Sx $\vee$ (((Qx) + (Rx)) $\cdot$ (Qy)) + (Ry) = S($x \oplus y$) $\vee$ $\neg$ ((Sx) $\cdot$ (Qy)) + (Ry) = S($x \oplus y$)

15: (((Qx) + (Rx)) $\cdot$ (Qy)) + (Ry) = S($x \oplus y$) by

2: (Qx) + (Rx) = Sx

14: $\neg$ (Qx) + (Rx) = Sx $\vee$ (((Qx) + (Rx)) $\cdot$ (Qy)) + (Ry) = S($x \oplus y$)

16: $\neg$ Qy is a power of two $\vee$ (Qx) $\cdot$ (Qy) is a power of two by

3: Qx is a power of two

7: $\neg$ Qx is a power of two $\vee$ $\neg$ Qy is a power of two $\vee$ (Qx) $\cdot$ (Qy) is a power of two

17: (Qx) $\cdot$ (Qy) is a power of two by

4: Qy is a power of two

16: $\neg$ Qy is a power of two $\vee$ (Qx) $\cdot$ (Qy) is a power of two

18: $\neg$ ((Qx) $\cdot$ (Qy)) + (((Rx) $\cdot$ (Qy)) + (Ry)) = (((Qx) $\cdot$ (Qy)) + ((Rx) $\cdot$ (Qy))) + (Ry) $\vee$ ((Qx) $\cdot$ (Qy)) + (((Rx) $\cdot$ (Qy)) + (Ry)) = (((Qx) + (Rx)) $\cdot$ (Qy)) + (Ry) by

5: ((Qx) $\cdot$ (Qy)) + ((Rx) $\cdot$ (Qy)) = ((Qx) + (Rx)) $\cdot$ (Qy)

12: $\neg$ ((Qx) $\cdot$ (Qy)) + ((Rx) $\cdot$ (Qy)) = ((Qx) + (Rx)) $\cdot$ (Qy) $\vee$ $\neg$ ((Qx) $\cdot$ (Qy)) + (((Rx) $\cdot$ (Qy)) + (Ry)) = (((Qx) $\cdot$ (Qy)) + ((Rx) $\cdot$ (Qy))) + (Ry) $\vee$ ((Qx) $\cdot$ (Qy)) + (((Rx) $\cdot$ (Qy)) + (Ry)) = (((Qx) + (Rx)) $\cdot$ (Qy)) + (Ry)

19: ((Qx) $\cdot$ (Qy)) + (((Rx) $\cdot$ (Qy)) + (Ry)) = (((Qx) + (Rx)) $\cdot$ (Qy)) + (Ry) by

6: ((Qx) $\cdot$ (Qy)) + (((Rx) $\cdot$ (Qy)) + (Ry)) = (((Qx) $\cdot$ (Qy)) + ((Rx) $\cdot$ (Qy))) + (Ry)

18: $\neg$ ((Qx) $\cdot$ (Qy)) + (((Rx) $\cdot$ (Qy)) + (Ry)) = (((Qx) $\cdot$ (Qy)) + ((Rx) $\cdot$ (Qy))) + (Ry) $\vee$ ((Qx) $\cdot$ (Qy)) + (((Rx) $\cdot$ (Qy)) + (Ry)) = (((Qx) + (Rx)) $\cdot$ (Qy)) + (Ry)

20: $\neg$ ((Qx) $\cdot$ (Qy)) + (((Rx) $\cdot$ (Qy)) + (Ry)) = S($x \oplus y$) $\vee$ $\neg$ (Qx) $\cdot$ (Qy) is a power of two $\vee$ (Qx) $\cdot$ (Qy) = Q($x \oplus y$) by

8: ((Rx) $\cdot$ (Qy)) + (Ry) $<$ (Qx) $\cdot$ (Qy)

9: $\neg$ ((Qx) $\cdot$ (Qy)) + (((Rx) $\cdot$ (Qy)) + (Ry)) = S($x \oplus y$) $\vee$ $\neg$ (Qx) $\cdot$ (Qy) is a power of two $\vee$ $\neg$ ((Rx) $\cdot$ (Qy)) + (Ry) $<$ (Qx) $\cdot$ (Qy) $\vee$ (Qx) $\cdot$ (Qy) = Q($x \oplus y$)

21: $\neg$ ((Qx) $\cdot$ (Qy)) + (((Rx) $\cdot$ (Qy)) + (Ry)) = S($x \oplus y$) $\vee$ $\neg$ (Qx) $\cdot$ (Qy) is a power of two $\vee$ ((Rx) $\cdot$ (Qy)) + (Ry) = R($x \oplus y$) by

8: $((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y) < (\mathrm{Q}x)\cdot(\mathrm{Q}y)$

10: $\neg\, ((\mathrm{Q}x)\cdot(\mathrm{Q}y)) + (((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y)) = \mathrm{S}(x\oplus y) \quad\vee\quad \neg\, (\mathrm{Q}x)\cdot(\mathrm{Q}y)$ is a power of two $\quad\vee\quad \neg\, ((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y) < (\mathrm{Q}x)\cdot(\mathrm{Q}y) \quad\vee\quad ((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y) = \mathrm{R}(x\oplus y)$

22: $\neg\, ((\mathrm{Q}x)\cdot(\mathrm{Q}y)) + (((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y)) = (((\mathrm{Q}x) + (\mathrm{R}x))\cdot(\mathrm{Q}y)) + (\mathrm{R}y) \quad\vee\quad ((\mathrm{Q}x)\cdot(\mathrm{Q}y)) + (((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y)) = \mathrm{S}(x\oplus y)$ by

15: $(((\mathrm{Q}x) + (\mathrm{R}x))\cdot(\mathrm{Q}y)) + (\mathrm{R}y) = \mathrm{S}(x\oplus y)$

13: $\neg\, ((\mathrm{Q}x)\cdot(\mathrm{Q}y)) + (((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y)) = (((\mathrm{Q}x) + (\mathrm{R}x))\cdot(\mathrm{Q}y)) + (\mathrm{R}y) \quad\vee\quad ((\mathrm{Q}x)\cdot(\mathrm{Q}y)) + (((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y)) = \mathrm{S}(x\oplus y) \quad\vee\quad \neg\, (((\mathrm{Q}x) + (\mathrm{R}x))\cdot(\mathrm{Q}y)) + (\mathrm{R}y) = \mathrm{S}(x\oplus y)$

23: $\neg\, ((\mathrm{Q}x)\cdot(\mathrm{Q}y)) + (((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y)) = \mathrm{S}(x\oplus y) \quad\vee\quad (\mathrm{Q}x)\cdot(\mathrm{Q}y) = \mathrm{Q}(x\oplus y)$ by

17: $(\mathrm{Q}x)\cdot(\mathrm{Q}y)$ is a power of two

20: $\neg\, ((\mathrm{Q}x)\cdot(\mathrm{Q}y)) + (((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y)) = \mathrm{S}(x\oplus y) \quad\vee\quad \neg\, (\mathrm{Q}x)\cdot(\mathrm{Q}y)$ is a power of two $\quad\vee\quad (\mathrm{Q}x)\cdot(\mathrm{Q}y) = \mathrm{Q}(x\oplus y)$

24: $\neg\, ((\mathrm{Q}x)\cdot(\mathrm{Q}y)) + (((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y)) = \mathrm{S}(x\oplus y) \quad\vee\quad ((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y) = \mathrm{R}(x\oplus y)$ by

17: $(\mathrm{Q}x)\cdot(\mathrm{Q}y)$ is a power of two

21: $\neg\, ((\mathrm{Q}x)\cdot(\mathrm{Q}y)) + (((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y)) = \mathrm{S}(x\oplus y) \quad\vee\quad \neg\, (\mathrm{Q}x)\cdot(\mathrm{Q}y)$ is a power of two $\quad\vee\quad ((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y) = \mathrm{R}(x\oplus y)$

25: $((\mathrm{Q}x)\cdot(\mathrm{Q}y)) + (((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y)) = \mathrm{S}(x\oplus y)$ by

19: $((\mathrm{Q}x)\cdot(\mathrm{Q}y)) + (((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y)) = (((\mathrm{Q}x) + (\mathrm{R}x))\cdot(\mathrm{Q}y)) + (\mathrm{R}y)$

22: $\neg\, ((\mathrm{Q}x)\cdot(\mathrm{Q}y)) + (((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y)) = (((\mathrm{Q}x) + (\mathrm{R}x))\cdot(\mathrm{Q}y)) + (\mathrm{R}y) \quad\vee\quad ((\mathrm{Q}x)\cdot(\mathrm{Q}y)) + (((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y)) = \mathrm{S}(x\oplus y)$

26: $(\mathrm{Q}x)\cdot(\mathrm{Q}y) = \mathrm{Q}(x\oplus y)$ by

25: $((\mathrm{Q}x)\cdot(\mathrm{Q}y)) + (((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y)) = \mathrm{S}(x\oplus y)$

23: $\neg\, ((\mathrm{Q}x)\cdot(\mathrm{Q}y)) + (((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y)) = \mathrm{S}(x\oplus y) \quad\vee\quad (\mathrm{Q}x)\cdot(\mathrm{Q}y) = \mathrm{Q}(x\oplus y)$

27: $((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y) = \mathrm{R}(x\oplus y)$ by

25: $((\mathrm{Q}x)\cdot(\mathrm{Q}y)) + (((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y)) = \mathrm{S}(x\oplus y)$

24: $\neg\, ((\mathrm{Q}x)\cdot(\mathrm{Q}y)) + (((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y)) = \mathrm{S}(x\oplus y) \quad\vee\quad ((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y) = \mathrm{R}(x\oplus y)$

28: $\neg\, ((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y) = \mathrm{R}(x\oplus y)$ by

26: $(\mathrm{Q}x)\cdot(\mathrm{Q}y) = \mathrm{Q}(x\oplus y)$

0: $\neg\, (\mathrm{Q}x)\cdot(\mathrm{Q}y) = \mathrm{Q}(x\oplus y) \quad\vee\quad \neg\, ((\mathrm{R}x)\cdot(\mathrm{Q}y)) + (\mathrm{R}y) = \mathrm{R}(x\oplus y)$

29: QEA by

27: $((\mathrm{R}x) \cdot (\mathrm{Q}y)) + (\mathrm{R}y) = \mathrm{R}(x \oplus y)$

28: $\neg\, ((\mathrm{R}x) \cdot (\mathrm{Q}y)) + (\mathrm{R}y) = \mathrm{R}(x \oplus y)$