Proof of Theorem 179

The theorem to be proved is

$S(x \oplus y) = Sx \cdot Qy + Ry$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\ (S(x \oplus y)) = (((Sx) \cdot (Qy)) + (Ry))]]$

Special cases of the hypothesis and previous results:

0: $\neg\ ((Sx) \cdot (Qy)) + (Ry) = S(x \oplus y)$ from H;x;y

1: $P(((Sx) \cdot (Qy)) + (Ry)) = x \oplus y$ from 172;x;y

2: $((Sx) \cdot (Qy)) + (Ry) = 0 \quad \vee \quad S(P(((Sx) \cdot (Qy)) + (Ry))) = ((Sx) \cdot (Qy)) + (Ry)$ from 22;$(Sx) \cdot (Qy)) + (Ry$

3: $\neg\ ((Sx) \cdot (Qy)) + (Ry) = 0 \quad \vee \quad (Sx) \cdot (Qy) = 0$ from 15;$Sx) \cdot (Qy$;Ry

4: $\neg\ Sx = 0$ from 3;x

5: $\neg\ Qy = 0$ from 178;y

6: $\neg\ (Sx) \cdot (Qy) = 0 \quad \vee \quad Sx = 0 \quad \vee \quad Qy = 0$ from 132;Sx;Qy

Equality substitutions:

7: $\neg\ P(((Sx) \cdot (Qy)) + (Ry)) = x \oplus y \quad \vee \quad \neg\ S(P(((Sx) \cdot (Qy)) + (Ry))) = ((Sx) \cdot (Qy)) + (Ry) \quad \vee \quad S(x \oplus y) = ((Sx) \cdot (Qy)) + (Ry)$

Inferences:

8: $\neg\ P(((Sx) \cdot (Qy)) + (Ry)) = x \oplus y \quad \vee \quad \neg\ S(P(((Sx) \cdot (Qy)) + (Ry))) = ((Sx) \cdot (Qy)) + (Ry)$ by

0: $\neg\ ((Sx) \cdot (Qy)) + (Ry) = S(x \oplus y)$

7: $\neg\ P(((Sx) \cdot (Qy)) + (Ry)) = x \oplus y \quad \vee \quad \neg\ S(P(((Sx) \cdot (Qy)) + (Ry))) = ((Sx) \cdot (Qy)) + (Ry) \quad \vee \quad ((Sx) \cdot (Qy)) + (Ry) = S(x \oplus y)$

9: $\neg\ S(P(((Sx) \cdot (Qy)) + (Ry))) = ((Sx) \cdot (Qy)) + (Ry)$ by

1: $P(((Sx) \cdot (Qy)) + (Ry)) = x \oplus y$

8: $\neg\ P(((Sx) \cdot (Qy)) + (Ry)) = x \oplus y \quad \vee \quad \neg\ S(P(((Sx) \cdot (Qy)) + (Ry))) = ((Sx) \cdot (Qy)) + (Ry)$

10: $\neg\ (\mathrm{S}x)\cdot(\mathrm{Q}y)=0 \quad \vee \quad \mathrm{Q}y=0$ by

4: $\neg\ \mathrm{S}x=0$

6: $\neg\ (\mathrm{S}x)\cdot(\mathrm{Q}y)=0 \quad \vee \quad \mathrm{S}x=0 \quad \vee \quad \mathrm{Q}y=0$

11: $\neg\ (\mathrm{S}x)\cdot(\mathrm{Q}y)=0$ by

5: $\neg\ \mathrm{Q}y=0$

10: $\neg\ (\mathrm{S}x)\cdot(\mathrm{Q}y)=0 \quad \vee \quad \mathrm{Q}y=0$

12: $((\mathrm{S}x)\cdot(\mathrm{Q}y))+(\mathrm{R}y)=0$ by

9: $\neg\ \mathrm{S}(\mathrm{P}(((\mathrm{S}x)\cdot(\mathrm{Q}y))+(\mathrm{R}y)))=((\mathrm{S}x)\cdot(\mathrm{Q}y))+(\mathrm{R}y)$

2: $((\mathrm{S}x)\cdot(\mathrm{Q}y))+(\mathrm{R}y)=0 \quad \vee \quad \mathrm{S}(\mathrm{P}(((\mathrm{S}x)\cdot(\mathrm{Q}y))+(\mathrm{R}y)))=((\mathrm{S}x)\cdot(\mathrm{Q}y))+(\mathrm{R}y)$

13: $\neg\ ((\mathrm{S}x)\cdot(\mathrm{Q}y))+(\mathrm{R}y)=0$ by

11: $\neg\ (\mathrm{S}x)\cdot(\mathrm{Q}y)=0$

3: $\neg\ ((\mathrm{S}x)\cdot(\mathrm{Q}y))+(\mathrm{R}y)=0 \quad \vee \quad (\mathrm{S}x)\cdot(\mathrm{Q}y)=0$

14: QEA by

12: $((\mathrm{S}x)\cdot(\mathrm{Q}y))+(\mathrm{R}y)=0$

13: $\neg\ ((\mathrm{S}x)\cdot(\mathrm{Q}y))+(\mathrm{R}y)=0$