Proof of Theorem 177

The theorem to be proved is

q_1 is a power of two & q_2 is a power of two → $q_1 \cdot q_2$ is a power of two

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) [[(q_1) is a power of two] & [(q_2) is a power of two] & [¬ ($q_1 \cdot q_2$) is a power of two]]

Special cases of the hypothesis and previous results:

0: q_1 is a power of two from H:q_1:q_2

1: q_2 is a power of two from H:q_1:q_2

2: ¬ $q_1 \cdot q_2$ is a power of two from H:q_1:q_2

3: ¬ q_1 is a power of two ∨ $2 \uparrow x_1 = q_1$ from 129$^{\rightarrow}$;q_1:x_1

4: ¬ q_2 is a power of two ∨ $2 \uparrow x_2 = q_2$ from 129$^{\rightarrow}$;q_2:x_2

5: $(2 \uparrow x_1) \cdot (2 \uparrow x_2) = 2 \uparrow (x_1 + x_2)$ from 136;2;x_1;x_2

6: $2 \uparrow (x_1 + x_2)$ is a power of two from 131;$x_1 + x_2$

Equality substitutions:

7: $\neg\, 2 \uparrow x_1 = q_1$ ∨ $\neg\, (2 \uparrow x_1) \cdot (2 \uparrow x_2) = 2 \uparrow (x_1 + x_2)$ ∨ $(q_1) \cdot (2 \uparrow x_2) = 2 \uparrow (x_1 + x_2)$

8: $\neg\, 2 \uparrow x_2 = q_2$ ∨ $\neg\, 2 \uparrow (x_1 + x_2) = q_1 \cdot (2 \uparrow x_2)$ ∨ $2 \uparrow (x_1 + x_2) = q_1 \cdot (q_2)$

9: $\neg\, 2 \uparrow (x_1 + x_2) = q_1 \cdot q_2$ ∨ ¬ $2 \uparrow (x_1 + x_2)$ is a power of two ∨ $q_1 \cdot q_2$ is a power of two

Inferences:

10: $2 \uparrow x_1 = q_1$ by
- 0: q_1 is a power of two
- 3: ¬ q_1 is a power of two ∨ $2 \uparrow x_1 = q_1$

11: $2 \uparrow x_2 = q_2$ by
- 1: q_2 is a power of two
- 4: ¬ q_2 is a power of two ∨ $2 \uparrow x_2 = q_2$

12: $\neg\, 2 \uparrow (x_1 + x_2) = q_1 \cdot q_2 \quad \vee \quad \neg\, 2 \uparrow (x_1 + x_2)$ is a power of two by

2: $\neg\, q_1 \cdot q_2$ is a power of two

9: $\neg\, 2 \uparrow (x_1 + x_2) = q_1 \cdot q_2 \quad \vee \quad \neg\, 2 \uparrow (x_1 + x_2)$ is a power of two $\vee$ $q_1 \cdot q_2$ is a power of two

13: $\neg\, 2 \uparrow x_1 = q_1 \quad \vee \quad 2 \uparrow (x_1 + x_2) = q_1 \cdot (2 \uparrow x_2)$ by

5: $(2 \uparrow x_1) \cdot (2 \uparrow x_2) = 2 \uparrow (x_1 + x_2)$

7: $\neg\, 2 \uparrow x_1 = q_1 \quad \vee \quad \neg\, (2 \uparrow x_1) \cdot (2 \uparrow x_2) = 2 \uparrow (x_1 + x_2) \quad \vee \quad 2 \uparrow (x_1 + x_2) = q_1 \cdot (2 \uparrow x_2)$

14: $\neg\, 2 \uparrow (x_1 + x_2) = q_1 \cdot q_2$ by

6: $2 \uparrow (x_1 + x_2)$ is a power of two

12: $\neg\, 2 \uparrow (x_1 + x_2) = q_1 \cdot q_2 \quad \vee \quad \neg\, 2 \uparrow (x_1 + x_2)$ is a power of two

15: $2 \uparrow (x_1 + x_2) = q_1 \cdot (2 \uparrow x_2)$ by

10: $2 \uparrow x_1 = q_1$

13: $\neg\, 2 \uparrow x_1 = q_1 \quad \vee \quad 2 \uparrow (x_1 + x_2) = q_1 \cdot (2 \uparrow x_2)$

16: $\neg\, 2 \uparrow (x_1 + x_2) = q_1 \cdot (2 \uparrow x_2) \quad \vee \quad 2 \uparrow (x_1 + x_2) = q_1 \cdot q_2$ by

11: $2 \uparrow x_2 = q_2$

8: $\neg\, 2 \uparrow x_2 = q_2 \quad \vee \quad \neg\, 2 \uparrow (x_1 + x_2) = q_1 \cdot (2 \uparrow x_2) \quad \vee \quad 2 \uparrow (x_1 + x_2) = q_1 \cdot q_2$

17: $\neg\, 2 \uparrow (x_1 + x_2) = q_1 \cdot (2 \uparrow x_2)$ by

14: $\neg\, 2 \uparrow (x_1 + x_2) = q_1 \cdot q_2$

16: $\neg\, 2 \uparrow (x_1 + x_2) = q_1 \cdot (2 \uparrow x_2) \quad \vee \quad 2 \uparrow (x_1 + x_2) = q_1 \cdot q_2$

18: QEA by

15: $2 \uparrow (x_1 + x_2) = q_1 \cdot (2 \uparrow x_2)$

17: $\neg\, 2 \uparrow (x_1 + x_2) = q_1 \cdot (2 \uparrow x_2)$