Proof of Theorem 176

The theorem to be proved is

$\mathrm{R}x \cdot \mathrm{Q}y + \mathrm{R}y < \mathrm{Q}x \cdot \mathrm{Q}y$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg (((\mathrm{R}x) \cdot (\mathrm{Q}y)) + (\mathrm{R}y)) < ((\mathrm{Q}x) \cdot (\mathrm{Q}y))]]$

Special cases of the hypothesis and previous results:

0: $\neg ((\mathrm{R}x) \cdot (\mathrm{Q}y)) + (\mathrm{R}y) < (\mathrm{Q}x) \cdot (\mathrm{Q}y)$ from H:x:y

1: $\mathrm{R}x < \mathrm{Q}x$ from 166;x

2: $\mathrm{R}y < \mathrm{Q}y$ from 166;y

3: $\neg \mathrm{R}x < \mathrm{Q}x \lor \neg \mathrm{R}y < \mathrm{Q}y \lor ((\mathrm{R}x) \cdot (\mathrm{Q}y)) + (\mathrm{R}y) < (\mathrm{Q}x) \cdot (\mathrm{Q}y)$ from 175;Rx;Qx;Ry;Qy

Inferences:

4: $\neg \mathrm{R}x < \mathrm{Q}x \lor \neg \mathrm{R}y < \mathrm{Q}y$ by
  0: $\neg ((\mathrm{R}x) \cdot (\mathrm{Q}y)) + (\mathrm{R}y) < (\mathrm{Q}x) \cdot (\mathrm{Q}y)$
  3: $\neg \mathrm{R}x < \mathrm{Q}x \lor \neg \mathrm{R}y < \mathrm{Q}y \lor ((\mathrm{R}x) \cdot (\mathrm{Q}y)) + (\mathrm{R}y) < (\mathrm{Q}x) \cdot (\mathrm{Q}y)$

5: $\neg \mathrm{R}y < \mathrm{Q}y$ by
  1: $\mathrm{R}x < \mathrm{Q}x$
  4: $\neg \mathrm{R}x < \mathrm{Q}x \lor \neg \mathrm{R}y < \mathrm{Q}y$

6: QEA by
  2: $\mathrm{R}y < \mathrm{Q}y$
  5: $\neg \mathrm{R}y < \mathrm{Q}y$