Proof of Theorem 166

The theorem to be proved is

$\mathrm{S}x = \mathrm{Q}x + \mathrm{R}x$ & $\mathrm{Q}x$ is a power of two & $\mathrm{R}x < \mathrm{Q}x$ $\star$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(\mathrm{S}x) = ((\mathrm{Q}x) + (\mathrm{R}x))$ $\vee$ $\neg\,(\mathrm{Q}x)$ is a power of two $\vee$ $\neg\,(\mathrm{R}x) < (\mathrm{Q}x)]]$

Special cases of the hypothesis and previous results:

0: $\neg\,(\mathrm{Q}x) + (\mathrm{R}x) = \mathrm{S}x$ $\vee$ $\neg\,\mathrm{Q}x$ is a power of two $\vee$ $\neg\,\mathrm{R}x < \mathrm{Q}x$ from H:x

1: $(\mathrm{Q}x) + (\mathrm{R}x) = \mathrm{S}x$ from 161;x

2: $\mathrm{Q}x$ is a power of two from 158;x

3: $\mathrm{R}x < \mathrm{Q}x$ from 165;x

Inferences:

4: $\neg\,\mathrm{Q}x$ is a power of two $\vee$ $\neg\,\mathrm{R}x < \mathrm{Q}x$ by
  1: $(\mathrm{Q}x) + (\mathrm{R}x) = \mathrm{S}x$
  0: $\neg\,(\mathrm{Q}x) + (\mathrm{R}x) = \mathrm{S}x$ $\vee$ $\neg\,\mathrm{Q}x$ is a power of two $\vee$ $\neg\,\mathrm{R}x < \mathrm{Q}x$

5: $\neg\,\mathrm{R}x < \mathrm{Q}x$ by
  2: $\mathrm{Q}x$ is a power of two
  4: $\neg\,\mathrm{Q}x$ is a power of two $\vee$ $\neg\,\mathrm{R}x < \mathrm{Q}x$

6: QEA by
  3: $\mathrm{R}x < \mathrm{Q}x$
  5: $\neg\,\mathrm{R}x < \mathrm{Q}x$