Proof of Theorem 164

The theorem to be proved is

$x + y < x + z \quad \to \quad y < z$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x + y) < (x + z)] \quad \& \quad [\neg\, (y) < (z)]]$

Special cases of the hypothesis and previous results:

0: $x + y < x + z$ from H:x:y:z

1: $\neg\, y < z$ from H:x:y:z

2: $\neg\, x + y < x + z \quad \vee \quad (x + y) + w = x + z$ from 162;$x + y$;$x + z$:w

3: $\neg\, x + y < x + z \quad \vee \quad \neg\, 0 = w$ from 162;$x + y$;$x + z$:w

4: $x + (y + w) = (x + y) + w$ from 72;x;y;w

5: $\neg\, x + (y + w) = x + z \quad \vee \quad y + w = z$ from 120;x;$y + w$;z

6: $\neg\, y + w = z \quad \vee \quad 0 = w \quad \vee \quad y < z$ from 163;y;w;z

Equality substitutions:

7: $\neg\, (x + y) + w = x + z \quad \vee \quad \neg\, x + (y + w) = (x + y) + w \quad \vee \quad x + (y + w) = x + z$

Inferences:

8: $(x + y) + w = x + z$ by
0: $x + y < x + z$
2: $\neg\, x + y < x + z \quad \vee \quad (x + y) + w = x + z$

9: $\neg\, 0 = w$ by
0: $x + y < x + z$
3: $\neg\, x + y < x + z \quad \vee \quad \neg\, 0 = w$

10: $\neg\, y + w = z \quad \vee \quad 0 = w$ by
1: $\neg\, y < z$
6: $\neg\, y + w = z \quad \vee \quad 0 = w \quad \vee \quad y < z$

11: $\neg\, (x + y) + w = x + z \quad \vee \quad x + (y + w) = x + z$ by
4: $x + (y + w) = (x + y) + w$
7: $\neg\, (x + y) + w = x + z \quad \vee \quad \neg\, x + (y + w) = (x + y) + w \quad \vee \quad x + (y + w) = x + z$

12: $x + (y + w) = x + z$ by

8: $(x + y) + w = x + z$

11: $\neg\, (x + y) + w = x + z \quad \vee \quad x + (y + w) = x + z$

13: $\neg\, y + w = z$ by

9: $\neg\, 0 = w$

10: $\neg\, y + w = z \quad \vee \quad 0 = w$

14: $y + w = z$ by

12: $x + (y + w) = x + z$

5: $\neg\, x + (y + w) = x + z \quad \vee \quad y + w = z$

15: QEA by

13: $\neg\, y + w = z$

14: $y + w = z$