Proof of Theorem 163

The theorem to be proved is

$x + y = z \quad \& \quad y \neq 0 \quad \to \quad x < z$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x + y) = (z)] \quad \& \quad [\neg\, (y) = (0)] \quad \& \quad [\neg\, (x) < (z)]]$

Special cases of the hypothesis and previous results:

0: $x + y = z$ from H:x:y:z

1: $\neg\, 0 = y$ from H:x:y:z

2: $\neg\, x < z$ from H:x:y:z

3: $x < z \quad \lor \quad \neg\, x \leq z \quad \lor \quad z = x$ from 56$^{\leftarrow}$;x;z

4: $x \leq x + y$ from 71;x;y

5: $x + 0 = x$ from 12;x

6: $\neg\, x + 0 = x + y \quad \lor \quad 0 = y$ from 120;x;y;0

Equality substitutions:

7: $\neg\, x + y = z \quad \lor \quad \neg\, x \leq x + y \quad \lor \quad x \leq z$

8: $\neg\, x + y = z \quad \lor \quad x + 0 = x + y \quad \lor \quad \neg\, x + 0 = z$

9: $\neg\, z = x \quad \lor \quad x + 0 = z \quad \lor \quad \neg\, x + 0 = x$

Inferences:

10: $\neg\, x \leq x + y \quad \lor \quad x \leq z$ by
0: $x + y = z$
7: $\neg\, x + y = z \quad \lor \quad \neg\, x \leq x + y \quad \lor \quad x \leq z$

11: $x + 0 = x + y \quad \lor \quad \neg\, x + 0 = z$ by
0: $x + y = z$
8: $\neg\, x + y = z \quad \lor \quad x + 0 = x + y \quad \lor \quad \neg\, x + 0 = z$

12: $\neg\, x + 0 = x + y$ by
1: $\neg\, 0 = y$
6: $\neg\, x + 0 = x + y \quad \lor \quad 0 = y$

13: $\neg\, x \leq z \quad \vee \quad z = x$ by

2: $\neg\, x < z$

3: $x < z \quad \vee \quad \neg\, x \leq z \quad \vee \quad z = x$

14: $x \leq z$ by

4: $x \leq x + y$

10: $\neg\, x \leq x + y \quad \vee \quad x \leq z$

15: $\neg\, z = x \quad \vee \quad x + 0 = z$ by

5: $x + 0 = x$

9: $\neg\, z = x \quad \vee \quad x + 0 = z \quad \vee \quad \neg\, x + 0 = x$

16: $\neg\, x + 0 = z$ by

12: $\neg\, x + 0 = x + y$

11: $x + 0 = x + y \quad \vee \quad \neg\, x + 0 = z$

17: $z = x$ by

14: $x \leq z$

13: $\neg\, x \leq z \quad \vee \quad z = x$

18: $\neg\, z = x$ by

16: $\neg\, x + 0 = z$

15: $\neg\, z = x \quad \vee \quad x + 0 = z$

19: QEA by

17: $z = x$

18: $\neg\, z = x$