Proof of Theorem 162

The theorem to be proved is

$x < y \quad \rightarrow \quad \exists w{\le}y[x + w = y \quad \& \quad w \neq 0]$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $\forall w[[(x) < (y)] \quad \& \quad [\neg\,(w) \le (y) \quad \vee \quad \neg\,(x + w) = (y) \quad \vee \quad (w) = (0)]]$

Special cases of the hypothesis and previous results:

0: $x < y$ from H:x;y;$y - x$

1: $\neg\, y - x \le y \quad \vee \quad \neg\, x + (y - x) = y \quad \vee \quad y - x = 0$ from H:x;y;$y - x$

2: $\neg\, x < y \quad \vee \quad x + (y - x) = y$ from 108;x;y

3: $\neg\, x < y \quad \vee \quad \neg\, y - x = 0$ from 108;x;y

4: $x + (y - x) = (y - x) + x$ from 98;x;$y - x$

5: $y - x \le (y - x) + x$ from 71;$y - x$;x

Equality substitutions:

6: $\neg\, x + (y - x) = y \quad \vee \quad \neg\, x + (y - x) = (y - x) + x \quad \vee \quad y = (y - x) + x$

7: $\neg\, (y - x) + x = y \quad \vee \quad \neg\, y - x \le (y - x) + x \quad \vee \quad y - x \le y$

Inferences:

8: $x + (y - x) = y$ by
0: $x < y$
2: $\neg\, x < y \quad \vee \quad x + (y - x) = y$

9: $\neg\, y - x = 0$ by
0: $x < y$
3: $\neg\, x < y \quad \vee \quad \neg\, y - x = 0$

10: $\neg\, x + (y - x) = y \quad \vee \quad (y - x) + x = y$ by
4: $x + (y - x) = (y - x) + x$
6: $\neg\, x + (y - x) = y \quad \vee \quad \neg\, x + (y - x) = (y - x) + x \quad \vee \quad (y - x) + x = y$

11: $\neg\,(y-x)+x=y \quad \vee \quad y-x\leq y$ by

5: $y-x\leq (y-x)+x$

7: $\neg\,(y-x)+x=y \quad \vee \quad \neg\, y-x\leq (y-x)+x \quad \vee \quad y-x\leq y$

12: $\neg\, y-x\leq y \quad \vee \quad y-x=0$ by

8: $x+(y-x)=y$

1: $\neg\, y-x\leq y \quad \vee \quad \neg\, x+(y-x)=y \quad \vee \quad y-x=0$

13: $(y-x)+x=y$ by

8: $x+(y-x)=y$

10: $\neg\, x+(y-x)=y \quad \vee \quad (y-x)+x=y$

14: $\neg\, y-x\leq y$ by

9: $\neg\, y-x=0$

12: $\neg\, y-x\leq y \quad \vee \quad y-x=0$

15: $y-x\leq y$ by

13: $(y-x)+x=y$

11: $\neg\,(y-x)+x=y \quad \vee \quad y-x\leq y$

16: QEA by

14: $\neg\, y-x\leq y$

15: $y-x\leq y$