Proof of Theorem 161

The theorem to be proved is

$\mathrm{S}x = \mathrm{Q}x + \mathrm{R}x$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, (\mathrm{S}x) = ((\mathrm{Q}x) + (\mathrm{R}x))]]$

Special cases of the hypothesis and previous results:

0: $\neg\, (\mathrm{Q}x) + (\mathrm{R}x) = \mathrm{S}x$ from H:x

1: $\mathrm{Q}x \leq \mathrm{S}x$ from 158;x

2: $(\mathrm{S}x) - (\mathrm{Q}x) = \mathrm{R}x$ from 160;x

3: $\neg\, \mathrm{Q}x \leq \mathrm{S}x \quad \vee \quad (\mathrm{Q}x) + ((\mathrm{S}x) - (\mathrm{Q}x)) = \mathrm{S}x$ from 68;Qx;Sx

Equality substitutions:

4: $\neg\, (\mathrm{S}x) - (\mathrm{Q}x) = \mathrm{R}x \quad \vee \quad \neg\, (\mathrm{Q}x) + ((\mathrm{S}x) - (\mathrm{Q}x)) = \mathrm{S}x \quad \vee \quad (\mathrm{Q}x) + (\mathrm{R}x) = \mathrm{S}x$

Inferences:

5: $\neg\, (\mathrm{S}x) - (\mathrm{Q}x) = \mathrm{R}x \quad \vee \quad \neg\, (\mathrm{Q}x) + ((\mathrm{S}x) - (\mathrm{Q}x)) = \mathrm{S}x$ by
0: $\neg\, (\mathrm{Q}x) + (\mathrm{R}x) = \mathrm{S}x$
4: $\neg\, (\mathrm{S}x) - (\mathrm{Q}x) = \mathrm{R}x \quad \vee \quad \neg\, (\mathrm{Q}x) + ((\mathrm{S}x) - (\mathrm{Q}x)) = \mathrm{S}x \quad \vee \quad (\mathrm{Q}x) + (\mathrm{R}x) = \mathrm{S}x$

6: $(\mathrm{Q}x) + ((\mathrm{S}x) - (\mathrm{Q}x)) = \mathrm{S}x$ by
1: $\mathrm{Q}x \leq \mathrm{S}x$
3: $\neg\, \mathrm{Q}x \leq \mathrm{S}x \quad \vee \quad (\mathrm{Q}x) + ((\mathrm{S}x) - (\mathrm{Q}x)) = \mathrm{S}x$

7: $\neg\, (\mathrm{Q}x) + ((\mathrm{S}x) - (\mathrm{Q}x)) = \mathrm{S}x$ by
2: $(\mathrm{S}x) - (\mathrm{Q}x) = \mathrm{R}x$
5: $\neg\, (\mathrm{S}x) - (\mathrm{Q}x) = \mathrm{R}x \quad \vee \quad \neg\, (\mathrm{Q}x) + ((\mathrm{S}x) - (\mathrm{Q}x)) = \mathrm{S}x$

8: QEA by
6: $(\mathrm{Q}x) + ((\mathrm{S}x) - (\mathrm{Q}x)) = \mathrm{S}x$
7: $\neg\, (\mathrm{Q}x) + ((\mathrm{S}x) - (\mathrm{Q}x)) = \mathrm{S}x$