Proof of Theorem 15b

The theorem to be proved is

$x + 0 = 0 \quad \rightarrow \quad x = 0 \quad \& \quad 0 = 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x + 0) = (0)] \quad \& \quad [\neg\, (x) = (0) \quad \vee \quad \neg\, (0) = (0)]]$

Special cases of the hypothesis and previous results:

0: $x + 0 = 0$ from H:x

1: $\neg\, 0 = x \quad \vee \quad \neg\, 0 = 0$ from H:x

2: $x + 0 = x$ from 12;x

Equality substitutions:

3: $\neg\, x + 0 = 0 \quad \vee \quad \neg\, x + 0 = x \quad \vee \quad 0 = x$

4: $\neg\, 0 = x \quad \vee \quad 0 = 0 \quad \vee \quad \neg\, x = x$

Inferences:

5: $\neg\, x + 0 = x \quad \vee \quad 0 = x$ by
0: $x + 0 = 0$
3: $\neg\, x + 0 = 0 \quad \vee \quad \neg\, x + 0 = x \quad \vee \quad 0 = x$

6: $0 = x$ by
2: $x + 0 = x$
5: $\neg\, x + 0 = x \quad \vee \quad 0 = x$

7: $\neg\, 0 = 0$ by
6: $0 = x$
1: $\neg\, 0 = x \quad \vee \quad \neg\, 0 = 0$

8: $0 = 0$ by
6: $0 = x$
4: $\neg\, 0 = x \quad \vee \quad 0 = 0$

9: QEA by
7: $\neg\, 0 = 0$
8: $0 = 0$