Proof of Theorem 159

The theorem to be proved is

q is a power of two $\quad \& \quad q \leq \mathrm{S}x \quad \& \quad \mathrm{S}x < 2 \cdot q \quad \rightarrow \quad q = \mathrm{Q}x$ $\qquad \star$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $\quad$ [[(q) is a power of two] $\quad \& \quad [(q) \leq (\mathrm{S}x)] \quad \& \quad [(\mathrm{S}x) < (2 \cdot q)] \quad \& \quad [\neg\, (q) = (\mathrm{Q}x)]]$

Special cases of the hypothesis and previous results:

0: q is a power of two $\quad$ from $\quad$ H:q:x

1: $q \leq \mathrm{S}x$ $\quad$ from $\quad$ H:q:x

2: $\mathrm{S}x < 2 \cdot q$ $\quad$ from $\quad$ H:q:x

3: $\neg\, \mathrm{Q}x = q$ $\quad$ from $\quad$ H:q:x

4: $\mathrm{p}_{150}((qx)) \quad \vee \quad \neg\, q$ is a power of two $\quad \vee \quad \neg\, q \leq \mathrm{S}x \quad \vee \quad \neg\, \mathrm{S}x < 2 \cdot q$ $\quad$ from 150$^{\leftarrow}$;q;x

5: $\mathrm{Q}x = q \quad \vee \quad \neg\, \mathrm{p}_{150}((qx))$ $\quad$ from $\quad$ 157$^{\leftarrow}$;x;q

Inferences:

6: $\mathrm{p}_{150}((qx)) \quad \vee \quad \neg\, q \leq \mathrm{S}x \quad \vee \quad \neg\, \mathrm{S}x < 2 \cdot q$ $\quad$ by
  0: q is a power of two
  4: $\mathrm{p}_{150}((qx)) \quad \vee \quad \neg\, q$ is a power of two $\quad \vee \quad \neg\, q \leq \mathrm{S}x \quad \vee \quad \neg\, \mathrm{S}x < 2 \cdot q$

7: $\mathrm{p}_{150}((qx)) \quad \vee \quad \neg\, \mathrm{S}x < 2 \cdot q$ $\quad$ by
  1: $q \leq \mathrm{S}x$
  6: $\mathrm{p}_{150}((qx)) \quad \vee \quad \neg\, q \leq \mathrm{S}x \quad \vee \quad \neg\, \mathrm{S}x < 2 \cdot q$

8: $\mathrm{p}_{150}((qx))$ $\quad$ by
  2: $\mathrm{S}x < 2 \cdot q$
  7: $\mathrm{p}_{150}((qx)) \quad \vee \quad \neg\, \mathrm{S}x < 2 \cdot q$

9: $\neg\, \mathrm{p}_{150}((qx))$ $\quad$ by
  3: $\neg\, \mathrm{Q}x = q$
  5: $\mathrm{Q}x = q \quad \vee \quad \neg\, \mathrm{p}_{150}((qx))$

10: QEA $\quad$ by
  8: $\mathrm{p}_{150}((qx))$
  9: $\neg\, \mathrm{p}_{150}((qx))$