Proof of Theorem 158

The theorem to be proved is

$\mathrm{Q}x$ is a power of two & $\mathrm{Q}x \leq \mathrm{S}x$ & $\mathrm{S}x < 2 \cdot \mathrm{Q}x$ ⋆

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, (\mathrm{Q}x)$ is a power of two $\vee$ $\neg\, (\mathrm{Q}x) \leq (\mathrm{S}x)$ $\vee$ $\neg\, (\mathrm{S}x) < (2 \cdot (\mathrm{Q}x))]]$

Special cases of the hypothesis and previous results:

0: $\neg\, \mathrm{Q}x$ is a power of two $\vee$ $\neg\, \mathrm{Q}x \leq \mathrm{S}x$ $\vee$ $\neg\, \mathrm{S}x < 2 \cdot (\mathrm{Q}x)$ from H:x

1: $\neg\, \mathrm{Q}x = \mathrm{Q}x$ $\vee$ $\mathrm{p}_{150}((\mathrm{Q}xx))$ from 157$^{\rightarrow}$;x;$\mathrm{Q}x$

2: $\neg\, \mathrm{p}_{150}((\mathrm{Q}xx))$ $\vee$ $\mathrm{Q}x$ is a power of two from 150$^{\rightarrow}$;$\mathrm{Q}x$;x

3: $\neg\, \mathrm{p}_{150}((\mathrm{Q}xx))$ $\vee$ $\mathrm{Q}x \leq \mathrm{S}x$ from 150$^{\rightarrow}$;$\mathrm{Q}x$;x

4: $\neg\, \mathrm{p}_{150}((\mathrm{Q}xx))$ $\vee$ $\mathrm{S}x < 2 \cdot (\mathrm{Q}x)$ from 150$^{\rightarrow}$;$\mathrm{Q}x$;x

5: $\mathrm{Q}x = \mathrm{Q}x$ from 5;$\mathrm{Q}x$

Inferences:

6: $\mathrm{p}_{150}((\mathrm{Q}xx))$ by
5: $\mathrm{Q}x = \mathrm{Q}x$
1: $\neg\, \mathrm{Q}x = \mathrm{Q}x$ $\vee$ $\mathrm{p}_{150}((\mathrm{Q}xx))$

7: $\mathrm{Q}x$ is a power of two by
6: $\mathrm{p}_{150}((\mathrm{Q}xx))$
2: $\neg\, \mathrm{p}_{150}((\mathrm{Q}xx))$ $\vee$ $\mathrm{Q}x$ is a power of two

8: $\mathrm{Q}x \leq \mathrm{S}x$ by
6: $\mathrm{p}_{150}((\mathrm{Q}xx))$
3: $\neg\, \mathrm{p}_{150}((\mathrm{Q}xx))$ $\vee$ $\mathrm{Q}x \leq \mathrm{S}x$

9: $\mathrm{S}x < 2 \cdot (\mathrm{Q}x)$ by
6: $\mathrm{p}_{150}((\mathrm{Q}xx))$
4: $\neg\, \mathrm{p}_{150}((\mathrm{Q}xx))$ $\vee$ $\mathrm{S}x < 2 \cdot (\mathrm{Q}x)$

10: $\neg\, \mathrm{Q}x \leq \mathrm{S}x$ $\vee$ $\neg\, \mathrm{S}x < 2 \cdot (\mathrm{Q}x)$ by
7: $\mathrm{Q}x$ is a power of two
0: $\neg\, \mathrm{Q}x$ is a power of two $\vee$ $\neg\, \mathrm{Q}x \leq \mathrm{S}x$ $\vee$ $\neg\, \mathrm{S}x < 2 \cdot (\mathrm{Q}x)$

11: $\neg\, \mathrm{S}x < 2 \cdot (\mathrm{Q}x)$ by

8: $\mathrm{Q}x \leq \mathrm{S}x$

10: $\neg\, \mathrm{Q}x \leq \mathrm{S}x \quad \vee \quad \neg\, \mathrm{S}x < 2 \cdot (\mathrm{Q}x)$

12: QEA by

9: $\mathrm{S}x < 2 \cdot (\mathrm{Q}x)$

11: $\neg\, \mathrm{S}x < 2 \cdot (\mathrm{Q}x)$