## Proof of Theorem 156b

The theorem to be proved is

$\exists q{\leq}1[\mathrm{p}_{150}(q,0)]$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $\forall q[[\neg\ (q) \leq (1) \quad \vee \quad \neg\ \mathrm{p}_{150}((q0))]]$

### Special cases of the hypothesis and previous results:

0: $\neg\ 1 \leq 1 \quad \vee \quad \neg\ \mathrm{p}_{150}((10))$ from H;1

1: $\mathrm{S}0 = 1$ from 115

2: $\mathrm{p}_{150}((10)) \quad \vee \quad \neg$ 1 is a power of two $\quad \vee \quad \neg\ 1 \leq \mathrm{S}0 \quad \vee \quad \neg\ \mathrm{S}0 < 2 \cdot 1$ from $150^{\leftarrow}$;1;0

3: 1 is a power of two from 130

4: $1 \leq 1$ from 60;1

5: $\neg\ \mathrm{S}0 = 0$ from 3;0

6: $1 = 0 \quad \vee \quad 1 < 2 \cdot 1$ from 137;1

### Equality substitutions:

7: $\neg\ \mathrm{S}0 = 1 \quad \vee \quad 1 \leq \mathrm{S}0 \quad \vee \quad \neg\ 1 \leq 1$

8: $\neg\ \mathrm{S}0 = 1 \quad \vee \quad \mathrm{S}0 < 2 \cdot 1 \quad \vee \quad \neg\ 1 < 2 \cdot 1$

9: $\neg\ \mathrm{S}0 = 1 \quad \vee \quad \mathrm{S}0 = 0 \quad \vee \quad \neg\ 1 = 0$

### Inferences:

10: $1 \leq \mathrm{S}0 \quad \vee \quad \neg\ 1 \leq 1$ by
1: $\mathrm{S}0 = 1$
7: $\neg\ \mathrm{S}0 = 1 \quad \vee \quad 1 \leq \mathrm{S}0 \quad \vee \quad \neg\ 1 \leq 1$

11: $\mathrm{S}0 < 2 \cdot 1 \quad \vee \quad \neg\ 1 < 2 \cdot 1$ by
1: $\mathrm{S}0 = 1$
8: $\neg\ \mathrm{S}0 = 1 \quad \vee \quad \mathrm{S}0 < 2 \cdot 1 \quad \vee \quad \neg\ 1 < 2 \cdot 1$

12: $S0 = 0 \quad \vee \quad \neg\, 1 = 0$ by

1: $S0 = 1$

9: $\neg\, S0 = 1 \quad \vee \quad S0 = 0 \quad \vee \quad \neg\, 1 = 0$

13: $p_{150}((10)) \quad \vee \quad \neg\, 1 \leq S0 \quad \vee \quad \neg\, S0 < 2 \cdot 1$ by

3: 1 is a power of two

2: $p_{150}((10)) \quad \vee \quad \neg$ 1 is a power of two $\quad \vee \quad \neg\, 1 \leq S0 \quad \vee \quad \neg\, S0 < 2 \cdot 1$

14: $\neg\, p_{150}((10))$ by

4: $1 \leq 1$

0: $\neg\, 1 \leq 1 \quad \vee \quad \neg\, p_{150}((10))$

15: $1 \leq S0$ by

4: $1 \leq 1$

10: $1 \leq S0 \quad \vee \quad \neg\, 1 \leq 1$

16: $\neg\, 1 = 0$ by

5: $\neg\, S0 = 0$

12: $S0 = 0 \quad \vee \quad \neg\, 1 = 0$

17: $\neg\, 1 \leq S0 \quad \vee \quad \neg\, S0 < 2 \cdot 1$ by

14: $\neg\, p_{150}((10))$

13: $p_{150}((10)) \quad \vee \quad \neg\, 1 \leq S0 \quad \vee \quad \neg\, S0 < 2 \cdot 1$

18: $\neg\, S0 < 2 \cdot 1$ by

15: $1 \leq S0$

17: $\neg\, 1 \leq S0 \quad \vee \quad \neg\, S0 < 2 \cdot 1$

19: $1 < 2 \cdot 1$ by

16: $\neg\, 1 = 0$

6: $1 = 0 \quad \vee \quad 1 < 2 \cdot 1$

20: $\neg\, 1 < 2 \cdot 1$ by

18: $\neg\, S0 < 2 \cdot 1$

11: $S0 < 2 \cdot 1 \quad \vee \quad \neg\, 1 < 2 \cdot 1$

21: $QEA$ by

19: $1 < 2 \cdot 1$

20: $\neg\, 1 < 2 \cdot 1$