Proof of Theorem 155

The theorem to be proved is

$p_{150}(q, x) \quad \to \quad \exists q_1 {\leq} SSx[p_{150}(q_1, Sx)]$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $\forall q_1[[p_{150}((qx))] \quad \& \quad [\neg\ (q_1) \leq (S(Sx)) \quad \vee \quad \neg\ p_{150}((q_1 Sx))]]$

Special cases of the hypothesis and previous results:

0: $p_{150}((qx))$ from H:q:x

^{A} $\forall q_1[[p_{150}((qx))] \quad \& \quad [\neg\ (q_1) \leq (S(Sx)) \quad \vee \quad \neg\ p_{150}((q_1 Sx))]]$ remnant from H:q:x

1: $\neg\ q \leq S(Sx) \quad \vee \quad \neg\ p_{150}((qSx))$ from ^{A};q

2: $\neg\ 2 \cdot q \leq S(Sx) \quad \vee \quad \neg\ p_{150}((2 \cdot qSx))$ from ^{A};$2 \cdot q$

3: $\neg\ p_{150}((qx)) \quad \vee \quad \neg\ S(Sx) < 2 \cdot q \quad \vee \quad p_{150}((qSx))$ from 152;q;x

4: $\neg\ p_{150}((qx)) \quad \vee \quad S(Sx) < 2 \cdot q \quad \vee \quad p_{150}((2 \cdot qSx))$ from 153;q;x

5: $\neg\ p_{150}((qx)) \quad \vee \quad q \leq Sx$ from $150^{\rightarrow}$;q;x

6: $\neg\ p_{150}((2 \cdot qSx)) \quad \vee \quad 2 \cdot q \leq S(Sx)$ from $150^{\rightarrow}$;$2 \cdot q$;Sx

7: $\neg\ q \leq Sx \quad \vee \quad q \leq S(Sx)$ from 154;q;Sx

Inferences:

8: $\neg\ S(Sx) < 2 \cdot q \quad \vee \quad p_{150}((qSx))$ by
0: $p_{150}((qx))$
3: $\neg\ p_{150}((qx)) \quad \vee \quad \neg\ S(Sx) < 2 \cdot q \quad \vee \quad p_{150}((qSx))$

9: $S(Sx) < 2 \cdot q \quad \vee \quad p_{150}((2 \cdot qSx))$ by
0: $p_{150}((qx))$
4: $\neg\ p_{150}((qx)) \quad \vee \quad S(Sx) < 2 \cdot q \quad \vee \quad p_{150}((2 \cdot qSx))$

10: $q \leq Sx$ by
0: $p_{150}((qx))$
5: $\neg\ p_{150}((qx)) \quad \vee \quad q \leq Sx$

11: $q \leq S(Sx)$ by
10: $q \leq Sx$
7: $\neg\ q \leq Sx \quad \vee \quad q \leq S(Sx)$

12: $\neg\, \mathrm{p}_{150}((q\mathrm{S}x))$ by

11: $q \leq \mathrm{S}(\mathrm{S}x)$

1: $\neg\, q \leq \mathrm{S}(\mathrm{S}x) \quad \vee \quad \neg\, \mathrm{p}_{150}((q\mathrm{S}x))$

13: $\neg\, \mathrm{S}(\mathrm{S}x) < 2 \cdot q$ by

12: $\neg\, \mathrm{p}_{150}((q\mathrm{S}x))$

8: $\neg\, \mathrm{S}(\mathrm{S}x) < 2 \cdot q \quad \vee \quad \mathrm{p}_{150}((q\mathrm{S}x))$

14: $\mathrm{p}_{150}((2 \cdot q\mathrm{S}x))$ by

13: $\neg\, \mathrm{S}(\mathrm{S}x) < 2 \cdot q$

9: $\mathrm{S}(\mathrm{S}x) < 2 \cdot q \quad \vee \quad \mathrm{p}_{150}((2 \cdot q\mathrm{S}x))$

15: $\neg\, 2 \cdot q \leq \mathrm{S}(\mathrm{S}x)$ by

14: $\mathrm{p}_{150}((2 \cdot q\mathrm{S}x))$

2: $\neg\, 2 \cdot q \leq \mathrm{S}(\mathrm{S}x) \quad \vee \quad \neg\, \mathrm{p}_{150}((2 \cdot q\mathrm{S}x))$

16: $2 \cdot q \leq \mathrm{S}(\mathrm{S}x)$ by

14: $\mathrm{p}_{150}((2 \cdot q\mathrm{S}x))$

6: $\neg\, \mathrm{p}_{150}((2 \cdot q\mathrm{S}x)) \quad \vee \quad 2 \cdot q \leq \mathrm{S}(\mathrm{S}x)$

17: QEA by

15: $\neg\, 2 \cdot q \leq \mathrm{S}(\mathrm{S}x)$

16: $2 \cdot q \leq \mathrm{S}(\mathrm{S}x)$